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   HYPOTHESES

   RESEARCH METHODOLOGY

   RESULTS & DISCUSSION

   FINDINGS

   RECOMMENDATIONS/SUGGESTIONS

   CONCLUSIONS

   SCOPE FOR FURTHER RESEARCH

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- Sharma T., Kwatra, G. (2008) Effectiveness of Social Advertising: A Study of Selected Campaigns, Corporate Social Responsibility, Edited by David Crowther & Nicholas Capaldi, Ashgate Research Companion to Corporate Social Responsibility, Chapter 15, pp 287-303.

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- Schemenner, R.W., Huber, J.C. and Cook, R.L. (1987), "Geographic Differences and the Location of New Manufacturing Facilities," Journal of Urban Economics, Vol. 21, No. 1, pp. 83-104.

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SOLUTION OF MULTICOLLINEARITY BY RIDGE REGRESSION

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ABSTRACT

Adequate attention is required to give on the presence of multicollinearity and its solution through some variants of ordinary least square (OLS). The traditional solution is to collect more observations or to drop one or more variables, which may often be impracticable in many situations. Hence, an attempt is made to squeeze out maximum information from whatever data one has at one's disposal. At first detect the presence of multicollinearity and remedial measures then can be applied to alleviate those. It is proposed that in face of multicollinearity one may use: Ridge Regression (RR), Principal Component Regression (PCR) or Generalized Inverse Regression (GIR). This paper axiomatically looks into ridge regression only to solve the problem of multicollinearity. Tychonoff (1943) proposed a regularization, which is known as 'Tikhonov Regularization' and used most commonly for ill-posed problems. In Statistics, this is known as RR. Hoerl and Kennard (H-K) proposed the technique of RR, which became popular tool with data analysis faced with a high degree of multicollinearity in the data. H-K (1970 a, b) have suggested adding a small positive quantity in the diagonal elements of the design matrix, $X'X$ before inverting it. It is intriguing to notice that this method was in frequent use since 1943. No wonder that Tychonoff published it in 1943 in Russian journal named 'Doklady Akademii Nauk SSSR'. However, it was the time when more and more qualities of RR came to light that the controversy arose about who should take the credit – Tychonoff who has been using it and published it in 1943 in Russian or H-K who published it in 1970 in English.

KEYWORDS
Multicollinearity, Ridge Regression, Biasing Parameter, Variance Inflation Factor and Tikhonov Regularization.

INTRODUCTION
Multicollinearity and singularity are issues derived from a correlation matrix with too high of correlation among explanatory variables. Multicollinearity is when variables are highly correlated and singularity is when the variables are perfectly correlated. In the presence of multicollinearity, the OLS estimate is not obtainable. Multicollinearity exposes the redundancy of variables and it can cause both logical and statistical problems. Logically, redundant variables weaken the analysis (except in the case of factor analysis), through reduction of degrees of freedom error.

The traditional solution is to collect more data or to drop one or more variables. Collecting more data may often be expensive or not practicable in many situations and to drop one or more variables from the model to alleviate the problem of multicollinearity may lead to the specification bias and hence the solution may be worse than the disease in certain situations. One may be interested to squeeze out maximum information from whatever data one has at one's disposal. This has motivated the researchers to the development of some very ingenious statistical methods namely ridge regression, principal component regression and generalized inverse regression. These could fruitfully be applied to solve the problem of multicollinearity. This paper looks into RR only to solve the problem of multicollinearity.

Tychonoff (1943) proposed $\hat{x} = (A'A + \Gamma'\Gamma)^{-1} A'\hat{\beta}$. This is popularly known as 'Tikhonov Regularization' (TR) and the most common used regularization of ill-posed problems. In Statistics, TR is also known as RR. H-K (1970 a, b) proposed the technique of RR and then suggested adding a small positive quantity in the diagonal elements of the design matrix, $X'X$ before inverting it. In other words, they proposed $\hat{\beta}_R = (X'X + kI)^{-1} X'Y$, where $\hat{\beta}_R$ is a ridge estimate of the parameter vector, β.

MULTICOLLINEARITY AND CONSEQUENCES
Multicollinearity may be a possible problem in study with two or more explanatory variables because they measure essentially the same thing. Klein (1960) argued rules of thumb that it is not necessarily a problem unless the intercorrelation is high relative to the overall degree of multiple correlation. Frisch (1934) was the first researcher to seriously study the multicollinearity problem and he defined the term 'multicollinearity'. In the presence of multicollinearity, the OLS estimate is not obtainable. Presence of multicollinearity brings $\hat{\beta}$ far away in Euclidian distance from the true vector (β) that means we have garbage the estimates. One of the basic implicit assumptions of the classical linear regression model $Y = X\beta + U$ is that there are no exact linear relations holding among the observed values of the explanatory variables. In this situation the parameter vector is not estimable. Thus, the least square estimation procedure breaks down.

In practice an exact linear relationship is highly improbable, but the general interdependence of economic phenomena may easily result in the appearance of approximate linear relationships in the explanatory variables. Johnson, Reimer and Rothrock (1973) resorted to a symptomatic definition: "Multicollinearity is the name given to general problem which arises where some or all of the explanatory variables in a relation are so highly correlated one with another that it becomes very difficult, if not impossible, to disentangle is their separate influence and obtain a reasonably precise estimate of their relative effects". Cicci and Tapley (1988) discussed, 'When normal matrix contains one or more very small eigenvalues, multicollinearity is said to exist and when one or more eigenvalues are precisely zero, the multicollinearity is said to be exact'. A measure that is specifically directed at X is the condition number of $X'X$, which is the square-root ratio of the largest characteristic root of $X'X$ (after scaling each column so that it has unit length) to the smallest. Greene (2002) found that a measure that is specifically directed at X is the condition number of $X'X$, is the square-root ratio of the largest characteristic root of $X'X$ (after scaling each column so that it has unit length) to the smallest.

If goal is simply to predict Y from a set of X variables, then multicollinearity is not a problem. The predictions will still be accurate, and overall R^2 (or adjusted R^2) quantifies how well the model predicts the Y values. If our goal is to understand how the various X variables impact Y, then multicollinearity is a big problem. The presence of multicollinearity has a number of potentially serious effects on the least squares estimates of the regression coefficients. This is true that multicollinearity does not destroy the property of minimum variance. But this does not mean that variance of an OLS estimator will necessarily be small (in relation to the value of the estimator) in any given samples. Multicollinearity may inflate all the variances and consequently deflate all the t-values. We must see what happens or is likely to happen in any sample.

For multicollinearity, $\lambda_p \rightarrow 0$ and MSE ($\hat{\beta}$) tends to infinity, $\hat{\beta}$ is subject to very large variance. Often this reveals the low values of the usual t-ratio whose denominator has the square root of the diagonal elements of $(X'X)^{-1}$. Marquardt termed it as variance inflation factor (VIF) and suggests a rule of thumb according to which VIF (i) = r^{ii} > 5 indicates harmful multicollinearity, where r^{ii} is the (i, j)th element of the inverse $(X'X)^{-1}$ in the standardized data. Farrar and Glauber (1967) first suggested looking at the values of r^{ii} to diagnose multicollinearity. Theil (1971) shows that $r^{ii} = \frac{1}{(1-R_i^2)\|x_i\|^2}$

Where $\|x_i\|^2 = x_i'x_i$ and R_i^2 is the squared multiple correlation coefficient when x_i is regressed on the remaining (p – 1) regressors.
For geometrically appealing discussion of the fact that nonorthogonality leads to wide confidence intervals for the regression coefficients. Another practical difficulty with the estimated Student's t value based on multicollinear data is that these values are highly unstable, and often change their sign and relative magnitudes with minor perturbations in the data.

Multicollinearity can also result in β_i appearing to "have the wrong sign", i.e., opposite to a priori expectations of the researcher. The exact causes of wrong signs may be many, and what appears to be a wrong sign may not even be wrong. Most regression practitioners know about the problem, even though it is not a well-defined problem, in a puristic sense. Visco (1978) showed omitting a variable with relatively low t-value couldn't correct the wrong sign of a regressor having a higher t-value.

Von Neumann (1941), Wilkinson (1965) and others developed the concept of stability of β_i vales with the use of some classical concepts in perturbation theory. Beaton, Rubin and Barone (1976) perturb the available data on GNP, unemployment, etc. by adding uniform random number from - 0.500 to + 0.499, observed that these minuscule variations drastically change most β_i (e.g. from - 232 to + 237) and conventional tests of significance do not provide adequate information about this kind of stability. In fact, Vinod (1981) showed the student's t values are themselves quite unstable.

DETECTION AND MEASUREMENT OF MULTICOLLINEARITY

Determination of the severity and form of near exact linear dependencies is an obvious initial step before any remedial measures. The existence and form of near exact linear dependencies should be measured.

If $|X'X|$ = 0, one or more exact linear dependencies exist among the columns of X and if $|X'X|$ = 1, the column of X are orthogonal. Farrar and Glauber (1967) defined a standard of comparison for $|X'X|$ by defining multicollinearity as a departure of the matrix from orthogonality. The estimates of both coefficient vector and its dispersion matrix require this operation. Using Wishart distribution, Wilks (1932) derives the moments & distribution of the determinant of the sample covariance matrix. He then obtained the moments and the distribution of determinants for sample correlation matrices as well. Explicit solutions for the distribution have not been obtained. However, Bartlett compares the lower moments of the Wilks distribution with those of chi-square distribution obtained a transformation of | R | as

$$\chi^2 = -[n - 1 - \tfrac{1}{6}(2p + 5)] \log |R|$$

That is distributed approximately as chi-square with $\frac{1}{2}$ p (p – 1) degrees of freedom, where n = size of the sample and p = number of variables. A high value of χ^2 indicates the existence of multicollinearity. Its severity can be measured by the level of significance at which the null hypothesis viz. H_0 : | R | = 1 is rejected. Cooley and Lohnes (1971) have completed a Monte Carlo study of this test.

Haitovsky (1969) gave a heuristic statistic, which is consistent with this concept. A small value of χ^2 indicates the existence of multicollinearity; its severity can be measured by the level of significance at which the null hypothesis H_0 : | R | = 0 is accepted.

Multicollinearity is essentially a problem of small | R |, and it is irrelevant what the specific elements of R are that produces | R |. More to the point, if $\lambda_1, \lambda_2, \ldots, \lambda_p$ are the eigenvalues of R (not necessarily distinct), then | R | = $\sum_{i=1}^{p} \lambda_i$. The small eigenvalues, therefore, result in small R. In fact, a singular matrix implies the existence of one or more zero eigenvalues. A rule can be established to constrain the smallest eigenvalue to be greater than a specific value. Dalling and Tamura (1970) suggested 0.3 as the specified value.

Klein suggests that the multicollinearity is said to be harmful if $|r_{ij}| > R_y$ for all i ≠ j, where r_{ij} is the zero order correlation between two predictor variables.

Farrar and Glauber found some drawbacks in Klein's rule and they developed a set of three tests for multicollinearity. The first test is based on χ^2 that has been discussed above. The second test is based on F test for locating which variables are multicollinear. Yet another test is a t test for finding out the pattern of multicollinearity, that is, for determining which variables are responsible for appearance of multicollinearity.

Compute the variance of regression coefficients $\sigma^2_{\hat{\beta}} = \sigma^2 (X'X)^{-1}$ and then compute variance based on standardized variables as $\sigma^2_{\hat{\beta}'} = (\sigma')^2 r_{xx}^{-1}$. The elements of the diagonal of the r_{xx}^{-1} matrix are the variance inflation factors (VIF). These are $VIF_i = (1 - R_i^2)^{-1}$. The value of VIF is unity when R_i^2 = 0 and this situation variable X_i is not correlated to other independent variables. The value of VIF is greater than unity in otherwise. The largest value of VIF (should not exceed 10) is an indicator of the multicollinearity. The mean of VIF is related to the severity of multicollinearity.
The Eigen values are extracted from the explanatory variables. These are variances of linear combinations of the explanatory variables. Now arrange these values in descending order of magnitude. If one or more, at the end, are zero then the matrix is not full rank. These sum to p, and if the X_i were independent,

each would equal to zero. The condition number is the square root of the ratio of the largest (always the first) to each of the others. If this value exceeds 30 then multicollinearity will be a problem.

RIDGE REGRESSION

The best solution is to understand the cause of multicollinearity and then apply RR, PCR or GIR. RR only could be one of the solutions of multicollinearity in this paper. Tychonoff (1943) discussed a regularization, which became popular as 'Tikhonov Regularization' (TR) and the most common used in case of ill-posed problems. He proposed $\hat{x} = (A'A + \Gamma'\Gamma)^{-1} A'\hat{\beta}$. TR has been invented independently in many different contexts. It became popular with its application to integral equations from the work of A. N. Tikhonov and D. L. Phillips. That is why some of authors call it 'Tikhonov-Phillps Regularization'. Hoerl expounded the finite dimensional case only by a statistical approach and it is known as RR. M. Foster interpreted TR as a Wiener-Kolmogorov filter. The regularization of the total least squares problem is based on TR and a generalized version of Tikhonov's method takes for the linear least square problem. Nair, Hegland and Anderssen (1997) applied TR to ill-posed operator equations. They proposed that assumptions must be made about the choice of the semi norm in the TR and the regularity of the least squares solutions, which one looks for. Qi-nian (1999) proposed a posteriori parameter choice strategy to choose the regularization parameter considering the finite-dimensional approximations of TR for nonlinear ill-posed problems with approximately given right-hand side. Saitoh (2007) used TR for a problem related to inverse problem. He introduced a general theory of TR using the theory of reproducing kernels including error estimates and convergence rates.

H-K (1970 a, b) proposed the technique of RR, which became a popular tool with data analysis faced with a high degree of multicollinearity in their data. They have suggested adding a small positive quantity in the diagonal elements of the design matrix, $X'X$ before inverting it, i.e., they proposed $\hat{\beta}_R = (X'X + kI)^{-1} X'Y$ in lieu of $\hat{\beta} = (X'X)^{-1} X'Y$. They showed that $\hat{\beta}_R$ has smaller mean square error than the OLS estimator, provided k is small enough and the standard regression model holds. Later, H-K (1976) published another popularized article that explored ridge regression as an approach to multiple linear regressions involving 'poorly-conditioned' data, that is, non-orthogonal predictor variable matrices. The genesis of RR lies with a paper written by Hoerl (1959) in which he discussed optimization from the response surface point of view. The next step in the development of ridge regression was the paper by Draper (1963), which provided the proofs lacking in Hoerl's paper. However, H-K (1970 a, b) developed a rigorous statistical basis for the application of ridge regression to the problem of multicollinearity in multiple linear regression models. Let $\hat{\beta}_R$ is the ridge estimate of the parameter vector, β in the linear model. Then

$$\hat{\beta}_R = (X'X + kI)^{-1} X'Y \qquad 0 < k < 1$$

RR was originally suggested as a procedure for investigating the sensitivity of least squares based on data exhibiting near extreme multicollinearity, where small perturbations in the data may produce large changes in the magnitude of the estimated coefficients. H-K (1970 a, b) introduce the generalized RR estimator (GRE) as

$$\hat{\beta}_{GR} = [X'X + PDP']^{-1} X'Y$$

Where P is the matrix whose columns are orthonormal characteristic vectors of $X'X$ and D is a diagonal matrix of constants $d_i \geq 0$.

If the constants d_i are all equal and take the value $d_i = k$, the GRE reduces to the RE as $\hat{\beta}_R = (X'X + kI)^{-1} X'Y$. The procedure of RE actually defines a family of estimators of which OLS estimator is a member for k = 0, i.e., with k = 0 the RE reduces to OLS estimator. The relation between OLS and RE estimators is as

$$[I + k(X'X)^{-1}]\hat{\beta}_R = \hat{\beta} \text{ and } V(\hat{\beta}_R) = \sigma^2 (X'X + kI)^{-1} X'X (X'X + kI)^{-1}, \text{ where}$$

$V(Y) = \sigma^2 I$ under the classical regression assumption.

Gauss (1809) suggested MSE as the most relevant criterion for choice among estimators. H-K (1970 a, b) gave theoretical justification for RR with the existence of strictly positive k as MSE ($\hat{\beta}_R$) < MSE ($\hat{\beta}$). Vinod (1976) proved for positive and stochastic k_i MSE ($\hat{\beta}_{GR}$) < MSE ($\hat{\beta}$) by interpreting Battacharya's (1966) estimator as a form of generalized RR. The estimated regression coefficients using RR and OLS are compared in terms of MSE, i. e., the average squared Euclidian distance between the estimate and the parameter. Hawkins (1975) outlined a technique named eigenanalysis and used as estimator, which is identical with RE.

The data matrix D = (Y: X) could be used to form another matrix T = D′D which could be diagonalized through an orthogonal matrix A such that ATA′ = diag (λ_i), where λ_i are eigenvalues of T. If a matrix T(k) = T + k I is obtained by suitably augmenting the data matrix with dummy observations, then since AT(k)A′ = diag (λ_i + k), a direct application of the result of Hawkins would lead to estimators which are identical with ridge estimate (RE). RR is closely related to the Bayesian estimation. Generally, if a p-variable normal distribution mean vector β_o and variance-covariance matrix V_o describes prior information about β, the Bayesian estimator of β is $\hat{\beta}_B = [\frac{1}{\sigma^2} X'X + V_o^{-1}]^{-1} [\frac{1}{\sigma^2} X'X + V_o^{-1} \beta]$. Many authors like Leamer (1978), Zellner (1971), etc. discussed the use of Bayesian method in regression. However, if we select prior mean $\beta_o = 0$ and $V_o = \sigma_0^2 I$, then $\hat{\beta}_B = (X'X + kI)^{-1} X'Y \equiv \hat{\beta}_R$ reduces to the usual ORE, when $k = \sigma^2/\sigma_0^2$.

CHOICE OF BIASING PARAMETER

Much controversy concerning RR centres on the choice of biasing parameter, k. Several authors have suggested methods for selecting the biasing parameter, k in the literatures. No firm recommendation to optimal choice of k seems to emerge. Some of them are as.

Wermuth (1972) suggested the value of k by minimizing the mean squared error of $\hat{\alpha}_R$ with respect to k.

$$k = \frac{\hat{\sigma}^2 \sum_{i=1}^{p} \lambda_i (\lambda_i + k)^{-3}}{\sum_{i=1}^{p} \lambda_i \hat{\alpha}_i^2 (\lambda_i + k)^{-3}}$$, where $\hat{\alpha} = P' \hat{\beta}$ and $\hat{\alpha}_R = P' \hat{\beta}_R$.

Dempster (1973) developed an empirical Bayes estimator for a prior distribution of α given as α ~ N (0, w^2 I). He then suggested replacing σ^2 by $\hat{\sigma}^2$ and the value of k is estimated by solving the equation.

$$p = \frac{\sum_{i=1}^{p} \hat{\alpha}_i^2}{\hat{\sigma}^2 (\frac{1}{k} + \frac{1}{\lambda_i})}$$

The ridge trace, as suggested by H-K (1970 a, b), is a two-dimensional plot of $\hat{\beta}_R$ against k in the interval [0, 1]. The second aspect is to determine the value of k, which gives a better estimate of β.

Theobald (1974) showed the difference of the MSE for $\hat{\beta}$ and $\hat{\beta}_R$ is a positive definite matrix, if k < $\frac{2\sigma^2}{\hat{\beta}' \hat{\beta}}$. Farebrother (1975) suggested the value of k that was empirically smaller optimal value of k than that of H-K (1970 a, b) and it is k = $\frac{\hat{\sigma}^2}{\hat{\beta}'\hat{\beta}}$. H-K and Baldwin (1975) suggested an appropriate choice for k as

$k_{HKB} = \frac{p\hat{\sigma}^2}{\hat{\beta}'\hat{\beta}}$, where $\hat{\beta}$ is OLS estimate of β, $\hat{\sigma}^2$ = (Y – Xβ)′ (Y – Xβ) and p is the number of explanatory variables.

Thisted (1976) modified the HKB estimator due to over shrinking towards zero and proposed k_T as an estimate.

$$k_T = \frac{(p-2)\hat{\sigma}^2}{\hat{\beta}'\hat{\beta}}$$

Schmidt (1976) suggested the value of k = $\frac{\sigma^2}{\max(\gamma_i)}$, which unfortunately gives a negative value of k. Vinod (1978) corrected it that is inefficient compared to Theobald's estimate.

In a subsequent paper H-K (1976) proposed an iterative estimation procedure based on k_{HKB} . They suggested the sequence of estimates of β and k.

$\hat{\beta}$: $k_0 = \frac{\hat{\sigma}^2}{\hat{\beta}'\hat{\beta}}$

$\hat{\beta}_R(k_0)$: $k_1 = \frac{p\hat{\sigma}^2}{\hat{\beta}_R(k_0)' \hat{\beta}_R(k_0)}$

$\hat{\beta}_R(k_1)$: $k_2 = \frac{p\hat{\sigma}^2}{\hat{\beta}_R(k_1)' \hat{\beta}_R(k_1)}$

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H-K proved superiority of RE over OLS estimator of a positive k whose range is 0 < k < $\frac{\sigma^2}{\gamma_0^2}$, where γ_0 is the largest element of vector γ = P' β.
Vinod (1976 b) advocated a modification of H-K's ridge trace (RT) method to make the optimal choice of biasing parameter more objective and meaningful. One suggestion of Vinod was to plot the regression estimates against m instead of k.

$$m = p - \sum_{i=1}^{p} \lambda_i (\lambda_i + k_i)^{-1}$$

The three main advantages are: (i) it can be used for generalized ridge regression (GRR) too, (ii) it narrows down the range of choice of k since 0 ≤ k ≤ ∞ corresponds to 1 ≤ m ≤ p, and (iii) it does not have the unfortunate property of Hoerl and Kennard's RT where even for completely orthogonal data the RT may appear more stable for larger value of k.
Ullah, Vinod and Kediyala (1981) suggested a family of double h-class ridge estimators, of which many earlier methods are special cases. They showed on the basis of MSE criterion the GRR would dominate over OLS estimate for

$k_i = h_1 \sigma^2 (r_i^2 - h_2 \sigma^2 \lambda_i^{-1})^{-1}$.

They further obtained a single value k as.

$k = p h_1 \hat{\sigma}^2 (C'C - h_2 \hat{\sigma}^2 \sum \lambda_i^{-1})^{-1}$ where C is the OLSE of vector Y.

This value of k coincides with those suggested by H-K & Baldwin (1975) and Farebrother (1975) for ($h_1 = 1$, $h_2 = 0$) and ($h_1 = \frac{1}{p}$ and $h_2 = 0$) respectively.

CONCLUDING REMARKS

Multicollinearity is suspected frequently for both a theoretical problem and problem with a particular sample of data. The problem of multicollinearity is as old as econometrics itself. At first detect the presence of multicollinearity in the data. Because of its presence in the data the design matrix becomes close to singular and hence the OLS estimate cannot be carried out. RR is an alternative estimation method when there is an extremely high degree of multicollinearity present in the data set (Darlington in 1978). RR is more advanced solution of multicollinearity and it injects small bias but generally greatly reduces the MSE giving more reliable estimates of β.

Singh (2010 b) discussed that a Russian article published by Tychonoff in 1943 became popular as 'Tikhonov Regularization' (TR). TR has been invented independently in many different contexts. It became widely known from its application to integral equations from the work of A. N. Tychonoff and D. L. Phillips. Some authors call the term Tikhonov-Phillips regularization. A. E. Hoerl expounded the finite dimensional case under statistical approach. M. Foster interpreted this method as a Wiener-Kolmogorov filter. Following Hoerl, it is known in the statistics literature as RR. TR is the most commonly used method of regularization of ill-posed problems and in Statistics, the method is also known as RR. It has relation to singular value decomposition and Wiener filter. Anders (2001) suggests that RR is an application of Tikhonov regularization, a method that has been explored in the approximation theory literature for about as long as RR has been used in Statistics.

The RE, different from OLS estimator, is a small positive increment (called biasing parameter) made to the diagonal element of the design matrix before inverting it. However, RE is biased; it has smaller mean square error than OLSE. RE is compared with other biased estimators. The ordinary ridge estimator with a given k is a biased linear estimator but value of $\hat{\beta}_R$ in certain interval has smaller mean square error than the OLS estimator. When the value of k is not given a prior and has to be determined from sample observations, the resulting OR estimators are no longer linear and can compete with OLS estimators on equal terms of the same prior information. Shapiro (1984) found that Vinod (1981) demonstrates estimated t-values are highly unstable with collinear data and often change their signs and relative magnitudes, even when the perturbations are minor. Vinod and Ullah (1981) found in general, any violations of the basic assumptions of least squares procedures tend to be more serious in the presence of multicollinearity.

McDonald and Schwing (1973) used the ridge regression procedure for analyzing mortality rates by various socio-economic (weather and pollution) variables. Vinod (1974) modified the canonical correlation analysis in the light of RR and used for estimating a joint production function.

In drawing conclusions one should remind of the fact that the assessment of the ORE and OLSE is based on entirely on the loss in estimation. Since the small sample properties of the non-linear ORE are not known, the ORR procedure is not suited for testing hypotheses. This makes ORR uninteresting for many econometricians and applied statisticians. It would be seen that the ORR might well become a powerful tool in forecasting, particularly in situations where a high degree of multicollinearity makes the OLS forecast unstable.

Although measure of credit is attributed to H-K for RR, which was published latter in the year 1970 in an English publication, widely circulated and read by the researchers. In contravention to the actual credit of Tychonoff for TR, this was published much earlier in the year 1943 in Russian language, which neither was widely circulated nor popular among the researchers. Moreover, it is appropriate to give higher credit to TR which gives findings of more general nature than RR by H-K which highly contextual in nature. Therefore, this controversy of attributing credit remains putting TR at disadvantage for lesser circulation in language of limited readership. Singh (2011) concluded that it is highly appropriate and legitimate to give credit to Tychonoff's TR which gave findings of more general than H-K's RR which highly contextual in nature. Singh (2010 a) found that it is similar to disadvantage born by Gauss who was profounder and user of 'Method of Least Squares' since 1794 without publishing whereas the credit was taken by Legendre who published the method much latter in 1805.

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