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FIXED POINT RESULTS FOR P-1 COMPATIBLE IN FUZZY MENGER SPACE

RUCHI SINGH
RESEARCH SCHOLAR
MVM GOVT SCIENCE COLLEGE
BHOPAL

A.D. SINGH
PROFESSOR
MVM GOVT SCIENCE COLLEGE
BHOPAL

ANIL GOYAL
PROFESSOR
UNIVERSITY INSTITUTE OF TECHNOLOGY RGPV
RAJEEV GANDHI TECHNICAL UNIVERSITY
BHOPAL

ABSTRACT

The study of theory of Fuzzy sets was initiated by Zadeh in 1965. Since then many authors have extended and developed the theory sets in the field of topology and analysis. The notion of Fuzzy metric spaces has very important applications in quantum particle physics. As a result many authors have extended the Banach's Contraction Principle to Fuzzy Menger Spaces and proved fixed point and common fixed point theorems on Fuzzy Menger Space. The aim of this paper is to prove common fixed point theorem in Fuzzy Menger Space for P-1 compatible mappings.

KEYWORDS

Fuzzy Menger Space, P-1 Compatible mappings, Common fixed point.

AMS SUBJECT CLASSIFICATION

47H10 and 54H24.

1. INTRODUCTION

Menger [5] in 1942 introduced the notation of the probabilistic metric space. The probabilistic generalization of metric space appears to be well adopted for the investigation of physical quantities and physiological thresholds. Schweizer and Sklar [7] studied this concept and then the important development of Menger space theory was due to Sehgal and Bharucha-Reid [8].Sessa [9] introduced weakly commuting maps in metric spaces. Jungck [2] enlarged this concept to compatible maps. The notion of compatible mapsin Menger spaces has been introduced by Mishra [6]. Cho [1] et al. and Sharma [10] gave fuzzy version of compatible maps and proved common fixed point theorems for compatible maps in fuzzy metric spaces. So many works have been done in fuzzy and Menger space [3],[4] and [12]. Sevet Kutukcu and Sushil Sharma introduce the concept of compatible maps of type (P-1) and type (P-2), show that they are equivalent to compatible maps under certain conditions and prove a common fixed point theorem for such maps in Menger spaces. Rajesh Shrivastav, Vivek Patel and Vanita Ben Dhagat[11] have given the definition of Fuzzy Menger space and proved fixed point theorem for such space. We prove fixed point results for Fuzzy Menger space with compatible P-1.

2. PRELIMINARIES

Definition 2.1 A fuzzy probabilistic metric space (FPM space) is an ordered pair (X,F_α) consisting of a nonempty set X and a mapping F_α from XxX into the collections of all fuzzy distribution functions $F_\alpha \in R$ for all $\alpha \in [0,1]$. For $x, y \in X$ we denote the fuzzy distribution function $F_\alpha(x,y)$ by $F_{\alpha(x,y)}$ and $F_{\alpha(x,y)}(u)$ is the value of $F_{\alpha(x,y)}$ at u in R.

The functions $F_{\alpha(x,y)}$ for all $\alpha \in [0,1]$ assumed to satisfy the following conditions:

(a) $F_{\alpha(x,y)}(u) = 1\ \forall\ u > 0$ iff $x = y$,
(b) $F_{\alpha(x,y)}(0) = 0\ \forall\ x, y$ in X,
(c) $F_{\alpha(x,y)} = F_{\alpha(y,x)}\ \forall\ x, y$ in X,
(d) If $F_{\alpha(x,y)}(u) = 1$ and $F_{\alpha(y,z)}(v) = 1 \Rightarrow F_{\alpha(x,z)}(u+v) = 1\ \forall\ x, y, z \in X$ and $u, v > 0$.

Definition 2.2 A commutative, associative and non-decreasing mapping t: $[0,1] \times [0,1] \longrightarrow [0,1]$ is a t-norm if and only if $t(a,1)= a\ \forall a \in [0,1]$, $t(0,0)=0$ and $t(c,d) \geq t(a,b)$ for $c \geq a$, $d \geq b$.

Definition 2.3 A Fuzzy Menger space is a triplet (X,F_α,t), where (X,F_α) is a FPM-space, t is a t-norm and the generalized triangle inequality

$F_{\alpha(x,z)}(u+v) \geq t(F_{\alpha(x,z)}(u), F_{\alpha(y,z)}(v))$

holds for all x, y, z in X u, v > 0 and $\alpha \in [0,1]$.

The concept of neighborhoods in Fuzzy Menger space is introduced as

Definition 2.4 Let (X,F_α,t) be a Fuzzy Menger space. If $x \in X$, $\varepsilon > 0$ and $\lambda \in (0,1)$, then (ε,λ) - neighborhood of x, called $U_x(\varepsilon,\lambda)$, is defined by $U_x(\varepsilon,\lambda) = \{y \in X: F_{\alpha(x,y)}(\varepsilon) > (1-\lambda)\}$.

An (ε,λ)-topology in X is the topology induced by the family $\{U_x(\varepsilon,\lambda): x \in X, \varepsilon > 0, \alpha \in [0,1]$ and $\lambda \in (0,1)\}$ of neighborhood.

Remark: If t is continuous, then Fuzzy Menger space (X,F_α,t) is a Housdroff space in (ε,λ)-topology.

Let (X,F_α,t) be a complete Fuzzy Menger space and $A \subset X$. Then A is called a bounded set if

$\lim_{u\to\infty} \inf_{x,y\in A} F_{\alpha(x,y)}(u) = 1$

Definition 2.5 A sequence $\{x_n\}$ in (X,F_α,t) is said to be convergent to a point x in X if for every $\varepsilon>0$and $\lambda>0$, there exists an integer $N=N(\varepsilon,\lambda)$ such that $x_n \in U_x(\varepsilon,\lambda)$ $\forall n \geq N$ or equivalently $F_\alpha(x_n, x; \varepsilon) > 1-\lambda$ for all $n \geq N$ and $\alpha \in [0,1]$.

Definition 2.6 A sequence $\{x_n\}$ in (X,F_α, t) is said to be cauchy sequence if for every $\varepsilon > 0$ and $\lambda > 0$, there exists an integer $N=N(\varepsilon,\lambda)$ such that for all $\alpha \in [0,1]$ $F_\alpha(x_n,x_m; \varepsilon) > 1-\lambda\ \forall\ n, m \geq N$.

Definition 2.7 A Fuzzy Menger space (X,F_α,t) with the continuous t-norm is said to be complete if every Cauchy sequence in X converges to a point in X for all $\alpha\in[0,1]$.

Following lemmas is selected from [8] , [12] and [13] respectively in fuzzy menger space.

Lemma 1 Let $\{x_n\}$ be a sequence in a Menger space $(X, F_\alpha,*)$ with continuous t-norm $*$ and $t * t \geq t$. If there exists a constant $k \in (0, 1)$ such that

$F_{\alpha(x_n,x_{n+1})}(kt) \geq F_{\alpha(x_{n-1},x_n)}(t)$ for all $t > 0$ and $n = 1, 2, \ldots$,

then $\{x_n\}$ is a Cauchy sequence in X.

Lemma 2 Let $(X, F_\alpha, *)$ be a Menger space. If there exists $k \in (0, 1)$ such that

$F_{\alpha(x,y)}(kt) \geq F_{\alpha(x,y)}(t)$ for all $x, y \in X$ and $t > 0$, then $x = y$.

Lemma 3. Let $\{y_n\}$ be a sequence in fuzzy Menger space $(X, F_\alpha,*)$ with continuous t-norm $*$ and $t * t \geq t$, for all $t \in [0,1]$ such that

$F_{\alpha(y_n,y_{n+1})}(kt) \geq \min\{F_{\alpha(y_{n-1},y_n)}(t), F_{\alpha(y_{n+1},y_n)}(t)\}$ for all $t > 0$ and $n \in N$.

Then $\{y_n\}$ is a Cauchy sequence in X.

Definition 2.8 Self maps A and B of a Fuzzy Menger space $(X, F_\alpha,*)$ are said to be compatible of type (P) if $F_{\alpha(ABxn,BBxn)}(t) \rightarrow 1$ and $F_{\alpha(BAxn,AAxn)}(t) \rightarrow 1$ $\forall t > 0$, whenever $\{x_n\}$ is a sequence in X such that $Ax_n,Bx_n \rightarrow z$ for some $z \in X$ as $n \rightarrow \infty$.

Definition 2.9 Self maps A and B of a Fuzzy Menger space $(X, F_\alpha, *)$ are said to be compatible of type (P-1) if $F_{\alpha(ABxnBBxn)}(t) \rightarrow 1$ for all $t > 0$, whenever $\{x_n\}$ is a sequence in X such that $Ax_n,Bx_n \rightarrow z$ for some z in X as $n \rightarrow \infty$.

3. Main Results

Theorem 3.1. Let A, B, S, T, L and M be self maps on a complete Fuzzy Menger space $(X, F_\alpha,*)$ with continuous $t-$norm $*$ defined as $a * b \geq \min(a,b)\,t$, for all $a,b \in [0,1]$,
satisfying:

(1.1) $AB(X) \subseteq M(X), ST(X) \subseteq L(X)$;

(1.2) $M(X)$ and $L(X)$ are complete subspace of X ;

(1.3) either AB or ST is continuous ;

(1.4) (AB, L) and (ST, M) are P-1 compatible;

(1.5) For all $x, y \in X, k \in (0,1), \beta \in (0,2), t > 0$,

$$F_{\alpha(ABx,STy)}(kt) \geq \min\{F_{\alpha(Lx,My)}(t), F_{\alpha(ABx,Lx)}(t), F_{\alpha(STy,My)}(t), F_{\alpha(ABx,My)}(\beta t), F_{\alpha(STy,Lx)}\big((2-\beta)t\big)\}$$

Then AB, ST, L and M have a unique common fixed point in X.

Proof. Let x_0 be an arbitrary point of X. Since $AB(X) \subseteq M(X), ST(X) \subseteq L(X)$ there exists $x_1, x_2 \in X$ such that $ABx_0 = Mx_1$ and $STx_1 = Lx_2$.

Inductively, we can construct sequences $\{x_n\}$ and $\{y_n\}$ in X such that

$$y_{2n-1} = Mx_{2n-1} = ABx_{2n-2} \text{ and } y_{2n} = Lx_{2n} = STx_{2n-1} \text{ for } n = 0,1,2,\ldots.$$

By taking $x = x_{2n}, y = x_{2n+1}$ and $\beta = 1 - q$ with $q \in (0,1)$ in (1.5), we have

$F_{\alpha(y_{2n+1},y_{2n+2})}(kt) = F_{\alpha(ABx_{2n}STQx_{2n+1})}(kt)$

$\geq \min\{F_{\alpha(Lx_{2n},Mx_{2n+1})}(t), F_{\alpha(ABx_{2n},Lx_{2n})}(t), F_{\alpha(ABx_{2n},Mx_{2n+1})}(\beta t), F_{\alpha(STx_{2n+1},Lx_{2n})}\big((2-\beta)t\big)\}$

$\geq \min\{F_{\alpha(y_{2n},y_{2n+1})}(t), F_{\alpha(y_{2n},y_{2n+1})}(t), F_{\alpha(y_{2n+2},y_{2n+1})}(t), F_{\alpha(y_{2n+1},y_{2n+1})}\big((1-q)t\big),$
$F_{\alpha(y_{2n+2},y_{2n})}\big((1+q)t\big)\}$

$\geq \min\{F_{\alpha(y_{2n},y_{2n+1})}(t), F_{\alpha(y_{2n},y_{2n+1})}(t), F_{\alpha(y_{2n+2},y_{2n+1})}(t), 1, F_{\alpha(y_{2n+2},y_{2n+1})}(t), F_{\alpha(y_{2n},y_{2n+1})}(qt)\}$

$\geq \min\{F_{\alpha(y_{2n},y_{2n+1})}(t), F_{\alpha(y_{2n+2},y_{2n+1})}(t), F_{\alpha(y_{2n},y_{2n+1})}(qt)\}$

Since $t-$norm is continuous, letting $q \rightarrow 1$, we have $\geq \min\{F_{\alpha(y_{2n},y_{2n+1})}(t), F_{\alpha(y_{2n+2},y_{2n+1})}(t), F_{\alpha(y_{2n},y_{2n+1})}(t)\}$

$\geq \min\{F_{\alpha(y_{2n},y_{2n+1})}(t), F_{\alpha(y_{2n+2},y_{2n+1})}(t)\}$

Thus we have $F_{\alpha(y_{2n+1},y_{2n+2})}(kt) \geq \min\{F_{\alpha(y_{2n},y_{2n+1})}(t), F_{\alpha(y_{2n+2},y_{2n+1})}(t)\}$

for $k \in (0,1)$ all $n \in N$ and $t > 0$. Hence, by Lemma 3, $\{y_n\}$ is a Cauchy sequence in X. Since $(X, F_\alpha,*)$ is complete, it converges to a point z in X. Also its subsequences converge to z.

Now, we prove z is the fixed point of AB, ST, Land M.

Case I. AB is continuous, (AB, L) and (ST, M) are compatible of type P-1.

Since AB is continuous, $AB(AB)x_{2n} \rightarrow ABz$ and $(AB)Lx_{2n} \rightarrow ABz$.

Since (AB, L) is compatible of type P-1, $(AB)Lx_{2n} \rightarrow ABz$.

By Uniqueness of limit in Menger space, we obtain $ABz = Lz$.

By taking $x = z$, $y = x_{2n+1}$ with $\beta= 1$ in (1.5), we have

$F_{\alpha(ABz,STx_{2n+1})}(kt) \geq \min\{F_{\alpha(Lz,Mx_{2n+1})}(t), F_{\alpha(ABz,Lz)}(t), F_{\alpha(STx_{2n+1},Mx_{2n+1})}(t),$
$F_{\alpha(STx_{2n+1},Lz)}(t), F_{\alpha(STx_{2n+1},Mx_{2n+1})}(t)\}$

This implies that, as $n \rightarrow \infty$

$F_{\alpha(ABz,z)}(kt) \geq \min\{F_{\alpha(Lz,z)}(t), F_{\alpha(ABz,Lz)}(t), F_{\alpha(z,z)}(t), F_{\alpha(z,Lz)}(t), F_{\alpha(z,z)}(t)\}$

$= \min\{F_{\alpha(Lz,z)}(t), 1, 1, F_{\alpha(z,Lz)}(t), 1\}$

$\geq F_{\alpha(Lz,z)}(t) = F_{\alpha(ABz,z)}(t)$

Thus by Lemma 2, it follows that $ABz = z$. Therefore, $z = ABz = Lz$.

Since $AB(X) \subset M(X)$, there exists $v \in X$ such that $z = ABz = Mv$. By taking $x = z, y = v$ with $\beta = 1$ in (1.5), we have

$$F_{\alpha(ABz,STv)}(kt) \geq \min\{F_{\alpha(Lz,Mv)}(t), F_{\alpha(ABz,Lz)}(t), F_{\alpha(STv,Mv)}(t), F_{\alpha(ABz,Mv)}(t), F_{\alpha(STv,Mv)}(t)\}$$

which implies that, as $n \rightarrow \infty$

$F_{\alpha(z,STv)}(kt) \geq \min\{F_{\alpha(z,z)}(t), F_{\alpha(z,z)}(t), F_{\alpha(STv,z)}(t), F_{\alpha(z,z)}(t), F_{\alpha(STv,z)}(t)\}$

$= \{1,1, F_{\alpha(STv,z)}(t), 1, F_{\alpha(STv,z)}(t)\}$

$\geq F_{\alpha(z,STv)}(t)$

Thus, by Lemma 2, we have $z = STv$.

Hence, $z = STv = Mv$.

As (ST, M) is compatible of type P-1, we have $ST(M)v = M(ST)v$.

Thus, $Mz = STz$.

By taking $x = z, y = z$ with $\beta = 1$ in (1.5), we get

$$F_{\alpha(ABz,STz)}(kt) \geq \min\{F_{\alpha(Lz,Mz)}(t), F_{\alpha(ABz,Lz)}(t), F_{\alpha(STz,Mz)}(t), F_{\alpha(ABz,Mz)}(t), F_{\alpha(STz,Lz)}(t)\}$$

which implies that, as $n \rightarrow \infty$

$F_{\alpha(ABz,STz)}(kt) \geq \min\{F_{\alpha(ABz,Mz)}(t), F_{\alpha(ABz,Lz)}(t), F_{\alpha(STz,Mz)}(t), F_{\alpha(ABz,Mz)}(t), F_{\alpha(STz,Lz)}(t)\}$

$$= \min\{ F_{\alpha(ABz,STz)}(t), F_{\alpha(Lz,Lz)}(t), F_{\alpha(Mz,Mz)}(t), F_{\alpha(ABz,STz)}(t,), F_{\alpha(STz,ABz)}(t)\}$$
$$= \min\{ F_{\alpha(ABz,STz)}(t), 1,1, F_{\alpha(ABz,STz)}(t), F_{\alpha(STz,ABz)}(t)\}$$
$$\geq F_{\alpha(ABz,STz)}(t)$$

Thus, by Lemma 2, we have $ABz = STz$. Therefore, $z = ABz = STz = Lz = Mz.z$

Thus z is the common fixed point of AB, ST, L, M.

Case II. ST is continuous, (AB, L) and (ST, M) are compatible of type P-1.

Since ST is continuous, $ST(ST)x_{2n} \to STz$ and $(ST)Mx_{2n} \to STz$.

Since $(ST, M)$is compatible of type P-1, $(ST)Mx_{2n} \to STz$.

By Uniqueness of limit in Menger space, we obtain $STz = Mz$

By taking $x = x_{2n+1}$and $y = z$ with β= 1 in (1.5), we have

$$F_{\alpha(ABx_{2n+1},STz)}(kt) \geq \min\{F_{\alpha(Lx_{2n+1},Mz)}(t), F_{\alpha(ABx_{2n+1},Lx_{2n+1})}(t), F_{\alpha(STz,Mz)}(t), F_{\alpha(STz,Lx_{2n+1})}(t), F_{\alpha(STz,Mz)}(t)\}$$

This implies that, as $n \to \infty$

$$F_{\alpha(z,STz)}(kt) \geq \min\{ F_{\alpha(z,Mz)}(t), F_{\alpha(z,z)}(t), F_{\alpha(STz,Mz)}(t), F_{\alpha(STz,z)}(t), F_{\alpha(STz,Mz)}(t)\}$$
$$= \min\{ F_{\alpha(z,STz)}(t), 1, F_{\alpha(STz,STz)}(t), F_{\alpha(STz,z)}(t), F_{\alpha(STz,STz)}(t)\}$$
$$= \min\{F_{\alpha(z,STz)}(t), 1, 1, F_{\alpha(STz,z)}(t), 1\} \geq F_{\alpha(z,STz)}(t)$$

Thus by Lemma 2, it follows that $STz = z$. Therefore, $z = STz = Mz$.

Since $ST(X) \subset L(X)$, there exists $v \in X$ such that $z = STz = Lv$.

By taking $x = v, y = z$ with $\beta = 1$ in (1.5), we have

$$F_{\alpha(ABv,STz)}(kt) \geq \min\{F_{\alpha(Lv,Mz)}(t), F_{\alpha(ABv,Lv)}(t), F_{\alpha(STz,Mz)}(t), F_{\alpha(ABv,Mz)}(t), F_{\alpha(STz,Mz)}(t)\}$$

which implies that, as $n \to \infty$

$$F_{\alpha(ABv,z)}(kt) \geq \min\{F_{\alpha(z,z)}(t), F_{\alpha(ABv,z)}(t), F_{\alpha(z,z)}(t), F_{\alpha(ABv,z)}(t), F_{\alpha(z,z)}(t)\}$$
$$= \min\{1, F_{\alpha(ABv,z)}(t), 1, 1, F_{\alpha(ABv,z)}(t)\} \geq F_{\alpha(ABv,z)}(t)$$

Thus, by Lemma 2, we have $z = ABv$.

Hence, $z = ABv = Lv$.

As(AB, L) is compatible of type P-1, we have $AB(L)v = L(AB)v$.

Thus, $Lz = ABz$.

By taking $x = z, y = z$ with $\beta = 1$ in (1.5), we get

$$F_{\alpha(ABz,STz)}(kt) \geq \min\{F_{\alpha(Lz,Mz)}(t), F_{\alpha(ABz,Lz)}(t), F_{\alpha(STz,Mz)}(t), F_{\alpha(ABz,Mz)}(t), F_{\alpha(STz,Lz)}(t)\}$$

which implies that, as $n \to \infty$

$$F_{\alpha(ABz,STz)}(kt) \geq \min\{ F_{\alpha(ABz,Mz)}(t), F_{\alpha(ABz,Lz)}(t), F_{\alpha(STz,Mz)}(t), F_{\alpha(ABz,Mz)}(t), F_{\alpha(STz,Lz)}(t)\}$$
$$= \min\{ F_{\alpha(ABz,STz)}(t), F_{\alpha(Lz,Lz)}(t), F_{\alpha(Mz,Mz)}(t), F_{\alpha(ABz,STz)}(t), F_{\alpha(STz,ABz)}(t)\}$$
$$= \min\{ F_{\alpha(ABz,STz)}(t), 1,1, F_{\alpha(ABz,STz)}(t), F_{\alpha(STz,ABz)}(t)\} \geq F_{\alpha(ABz,STz)}(t)$$

Thus, by Lemma 2, we have $ABz = STz$. Therefore, $z = ABz = STz = Lz = Mz.z$

Thus z is the common fixed point of AB, ST, L, M.

Uniqueness : Let w $(w \neq z)$be the another common fixed point of AB, ST, L and M,

Then $w = ABw = STw = Lw = Mw$,

By taking $x = z$ and $y = w$ in (1.5),we get

$$F_{\alpha(ABz,STw)}(kt) \geq \min\{F_{\alpha(Lz,Mw)}(t), F_{\alpha(ABz,Lz)}(t), F_{\alpha(STw,Mw)}(t), F_{\alpha(ABz,Mw)}(t), F_{\alpha(STw,Lz)}(t)\}$$

From above results, we have

$$F_{\alpha(z,w)}(kt) \geq \min\{F_{\alpha(z,w)}(t), F_{\alpha(z,z)}(t), F_{\alpha(w,w)}(t), F_{\alpha(z,w)}(t), F_{\alpha(w,z)}(t)\}$$
$$= \min\{F_{\alpha(z,w)}(t), 1,1, F_{\alpha(z,w)}(t), F_{\alpha(w,z)}(t)\} \geq F_{\alpha(z,w)}(t)$$

Hence, $z = w$ for all $x, y \in X$ and $t > 0$. Therefore z is the unique common fixed point of AB, ST, and M.

On taking $B = T = I$(identity maps) in above Theorem3.1 then we have the following :

Corollary 3.2: Let A, S, L and M be self maps on a complete Fuzzy Menger space $(X, F_\alpha, *)$ with continuous $t - \text{norm} *$ defined as $a * b \geq \min(a, b)\, t$, for all $a, b \in [0, 1]$,
satisfying:

(1.6) $A(X) \subseteq M(X), S(X) \subseteq L(X)$;
(1.7) $M(X)$and $L(X)$are complete subspace of X ;
(1.8) either A or S is continuous ;
(1.9) $(A, L)$and (S, M) are compatible of type P-1;
(1.10) For all $x, y \in X, k \in (0, 1), \beta \in (0, 2), t > 0$,

$$F_{\alpha(Ax,Sy)}(kt) \geq \min\{F_{\alpha(Lx,My)}(t), F_{\alpha(Ax,Lx)}(t), F_{\alpha(Sy,My)}(t), F_{\alpha(Bx,My)}(\beta t), F_{\alpha(Sy,Lx)}((2-\beta)t)\}$$

Then A, S, L and M have a unique common fixed point in X.

If we take A= $S, L = M$and $B = T = I$(identity maps) in above … …. Theorem ..then we have the following :

Corollary 3.3: Let A and L be self maps on a complete Fuzzy Menger space $(X, F_\alpha, *)$ with continuous $t - \text{norm} *$ defined as $a * b \geq \min(a, b)\, t$, for all $a, b \in [0, 1]$,
satisfying:

(1.11) $A(X) \subseteq L(X)$;
(1.12) $L(X)$are complete subspace of X ;
(1.13) L is continuous ;
(1.14) (A, L) is compatible of type P-1;
(1.15) For all $x, y \in X, k \in (0, 1), \beta \in (0, 2), t > 0$,

$$F_{\alpha(Ax,Ay)}(kt) \geq \min\{F_{\alpha(Lx,Ly)}(t), F_{\alpha(Ax,Lx)}(t), F_{\alpha(Ay,Ly)}(t), F_{\alpha(Ax,Ly)}(\beta t), F_{\alpha(Ay,Lx)}((2-\beta)t)\}$$

Then A and L have a unique common fixed point in X.

CONCLUSION

Fuzzy set theory and Fuzzy Fixed Point Theory has numerous applications in applied sciences and engineering such as neural network theory, stability theory, mathematical programming, modeling theory, medical sciences (medical genetics, nervous system), image processing, control theory, communications etc. As a result fuzzy fixed point theory has become an area of interest for specialists in fixed point theory. In this paper we have proved common fixed point theorems for some self mappings on Fuzzy Manger spaces with compatibility P-1.

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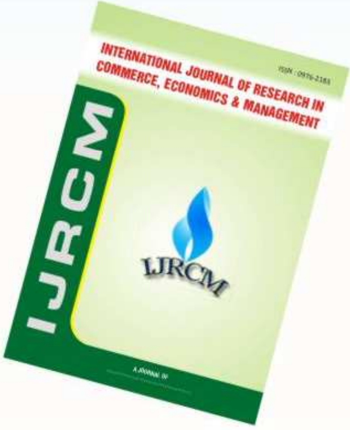
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