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INVENTORY MODELS FOR DETERIORATING ITEMS WITH STOCK DEPENDENT PRODUCTION AND DEMAND RATES HAVING WEIBULL DECAY

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ABSTRACT

In this paper we develop and analyze an inventory model for deteriorating items with the assumption that both production and demand rates are dependent on stock on-hand. It is further assumed that lifetime of the commodity is random and follows a three parameter Weibull distribution. Using the differential equations the instantaneous state of inventory is derived. With suitable cost considerations the total cost function is obtained. By minimizing the total cost function the optimal ordering policies are derived. Through numerical illustrations the sensitivity analysis is carried. It is observed that the stock dependent production parameters and the deterioration distribution parameters have significant influence on optimal production scheduling and total cost. This model also includes some of the earlier models as particular cases for specific values of the parameters. This model is much useful in analyzing several production processes.

KEYWORDS

EPQ model, stock dependent production, stock dependent demand, Weibull decay, production schedule.

1. INTRODUCTION

Inventory models are mathematical models which help a business firm to make optimal decisions as when and how much to buy (or produce) so as to minimize its cost or maximize its profit. Since the first classical lot-size formula was developed by F.W. Harris in 1915, a wide variety of inventory models have been developed and analyzed with various assumptions to deal with the real life situations. The efficiency of an inventory model depends upon the suitable assumptions made on the constituent components of the mode. The constituent components of the model are (1) replenishment (2) demand pattern and (3) lifetime of the commodity.

The implicit assumption of the Economic Order Quantity (EOQ) model is that the items are obtained from the outside supplier. Given this assumption, it is reasonable to assume that the entire lot is delivered at an instant of time. This means it is assumed that the replenishment rate is infinite. However, in some other places like production processes, manufacturing units, warehouses, the replenishment (production) is not instantaneous and may have finite rate. In the study of inventory models, economical production quantity (EPQ) model plays an important role. An EPQ model is an inventory model that determines the quantity of product to be produced on a single facility so as to meet the demand over an infinite planning horizon. The two most important decision variables in any EPQ model are the production run time and the optimal quantity to be produced. Production inventory models serve to find the optimal values of these decision variables while minimizing the total cost of production.

Acting as the driving force of the whole inventory system, demand is a key factor that should be taken into consideration in studying inventory systems. The demand for an item may be deterministic or stochastic. Demand may further be assumed to be constant over time, or variable depending on time, selling price of the item, amount of stock on hand, quality of the product or any other factors.

Decay or deterioration of physical goods while in stock is a common phenomenon. Pharmaceuticals, foods, vegetables and fruits are a few examples of such items. Taking this into consideration, deteriorating inventory models have been widely studied in recent years. Ghare and Schrader (1963) developed an economic order quantity model with constant rate of decay. Covert and Philip (1973) extended this model and obtained an economic order quantity model for a variable rate of deterioration by assuming a two-parameter Weibull distribution. Comprehensive reviews of research literature on deteriorating items are provided by Raafat (1991), Goyal and Giri (2001), Li, *et al*. (2010), Pentico and Drake (2011) and Bakker, *et al.* (2012).

In classical inventory models the demand rate is assumed to be constant. In reality, it may vary with time or with price or with the instantaneous level of inventory displayed in a supermarket. Manna, *et al.* (2007), Patra (2010), Teng, *et al.* (2011) studied economic order quantity models for deteriorating items when demand is quadratic in time. Skouri, *et al.* (2009) developed an order level inventory model with general ramp type demand rate, Weibull deterioration rate and partial backlogging of unsatisfied demand where the backlogging rate is any non-increasing function of the waiting time up to the next replenishment. Panda, *et al.* (2009), Panda and Saha (2010) and Manna and Chiang (2010) studied production inventory models having time-dependent demand and finite rate of production which is proportional to the demand rate.

It has been observed that for certain types of inventories, particularly consumer goods, heaps of stock will attract customers and the demand is a function of stock on-hand. Due to this fact, this area of inventory theory research has recently been receiving considerable attention. Mandal and Phaujdar (1989),Urban (1992), Giri *et al.* (1996), Zhou and Yang (2003), Teng *et al.* (2005), Uma Maheswara Rao, *et al* (2010), Yang, *et al.* (2010a) and others have developed inventory models where demand rate is a function of on-hand inventory.

In some production units dealing with food processing demand is a function of selling price. Maiti, *et al.* (2009) and Sana (2011) developed a finite time-horizon deterministic EOQ model where the rate of demand decreases quadratically with selling price. Widyadana and Wee (2012) developed an EPQ model for deteriorating items with rework. Production, rework, deteriorating and demand rates are assumed constant. Srinivasa Rao, *et. al* (2011) developed a production inventory system with demand rate a function of production quantity.

Inventory models for deteriorating items having multivariate demand functions were also studied by several authors. Chang, *et al*. (2010), Khanra, *et al.* (2010) considered selling price and stock level dependent demand rate and Uma Maheswara Rao, *et al.* (2010b) considered time and stock level dependent demand rate. Shah and Pandey (2009) developed a model for deteriorating items where demand is a function of frequency of advertisement and displayed inventory level by assuming infinite replenishment rate and Weibull rate of deterioration. Pal *et al.* (2006) considered a single deteriorating item with the demand rate dependent on displayed stock level, selling price of an item and frequency of advertisement.

But these authors have considered that the replenishment is instantaneous or have a fixed finite rate. Bhunia and Maiti (1998) developed inventory model without shortages for non deteriorating items by assuming production rate is linearly dependent on on-hand inventory and demand rate on time. An inventory model with fuzzy replenishment where demand rate is taken as a ramp type function of time was considered by Mahta and Goswami (2009). Sridevi *et al.* (2010) developed an inventory model for deteriorating items with random replenishment. Muluneh and Srinivasa Rao (2012) developed an inventory with the assumption that the production rate is dependent on stock on hand and demand is a power function of time. Recently, Muluneh and Srinivasa Rao (2013) developed an inventory model for deteriorating items with stock dependent production rate selling price dependent demand.

In this paper an inventory model for deteriorating items is developed with the assumption that both the replenishment and demand are functions of on-hand inventory. It is also assumed that the lifetime of the commodity is random and follows a three parameter Weibull distribution. The three parameter Weibull decay includes increasing/decreasing/ constant rates of decay. It also includes exponential decay as a particular case. The location parameter in the decay distribution characterizes the delayed decay of deteriorating items. The demand function considered here also includes the constant demand as a particular case. Assuming that shortages are allowed and fully backlogged, the instantaneous state of inventory is derived through differential equations. Production schedule and production quantity are derived by minimizing the total cost function. The sensitivity of the model with respect to the parameters and cost is also studied. This model is extended to the case of without shortages.

2. NOTATIONS AND ASSUMPTIONS

2.1. NOTATIONS

The following notations are used in developing the model.

I(t)	inventory level at any time *t*
A	set up cost
c_2	shortage cost per unit per unit time
h	inventory holding cost per unit per unit time
H	total holding cost in a cycle time
p	per unit production cost (cost price) of the item
S_h	total shortage cost in a cycle time
D(t)	Rate of demand at any time *t*
R(t)	rate of production at any time t
Q	production quantity
T	production cycle time
TC	total production cost per unit time
(α, β, γ)	deterioration rate parameters
(τ, ϕ)	demand rate parameters
(ϕ, θ)	production rate parameters

2.2. ASSUMPTIONS

The model is developed based on the following assumptions.

i. Lifetime of the commodity is random and follows a three parameter Weibull distribution with probability density function of the form

$$f(t)=\alpha\beta(t-\gamma)^{\beta-1}e^{-\alpha(t-\gamma)^{\beta}}, \quad \alpha,\beta>0,\ t\ge\gamma$$

where, α is scale parameter, β is shape parameter and γ is location parameter.

The instantaneous rate of deterioration at time *t* is

$$h(t)=\frac{f(t)}{1-F(t)}=\alpha\beta(t-\gamma)^{\beta-1}$$

ii. The demand rate function *D(t)* is a known function of instantaneous stock level, *I(t).* Its functional form is given by

$$D(t)=\begin{cases}\tau+\varphi I(t), & 0\le t\le t_2\\ \tau, & t_2\le t\le T\end{cases}$$

where, τ is a positive constant and φ is the stock-dependent consumption rate parameter, $0\le\varphi\le 1$. If $\varphi=0$, then the demand rate is constant.

iii. Production rate is assumed to be finite and a decreasing linear function of the instantaneous stock level inventory level, *I(t),* i.e.,

$$R(t)=\begin{cases}\theta-\phi I(t), & 0\le t\le t_2\\ \theta, & t_2\le t\le T\end{cases}$$

where, $\theta\ge 0$, φ is the stock-dependent production rate parameter, $0\le\phi\le 1$ and it is assumed that $R(t)\ge D(t)$ at any time where replenishment takes place.

If ϕ=0, then it includes the finite rate of production.

iv. There is no repair or replacement of deteriorated items.

v. The planning horizon is infinite. Each cycle will have length *T*.

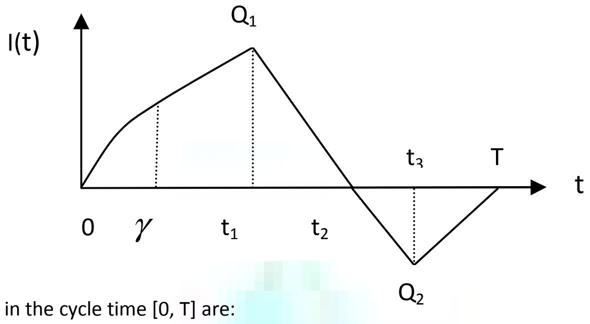
vi. The inventory holding cost per unit per unit time *h*, the shortage cost per unit per unit time c_2 and the unit production cost per unit time *p* and set up cost *A* per cycle are fixed and known,

3. INVENTORY MODEL WITH SHORTAGES

3.1. MODEL FORMULATION

Consider an inventory system for deteriorating items in which the lifetime of the commodity is random and follows a three parameter Weibull distribution. Here it is assumed that shortages are allowed and fully backlogged. In this model the stock level for the item is initially zero. Then production starts at time *t=0*, and continues adding items to stock until the on-hand inventory reaches its maximum level Q_1 at time $t=t_1$. During the time $(0,\gamma)$ demand is met from replenishment and the remaining will be accumulated in stock. At time $t=\gamma$ deterioration of the item starts and stock is depleted by demand and deterioration while production is continuously adding to it. At $t=t_1$ production is stopped and stock will be depleted by deterioration and demand until it reaches zero at time $t=t_2$. As demand is assumed to occur continuously, at this point shortages begin to accumulate until it reaches its maximum level of Q_2 at $t=t_3$. At this point production will resume meeting the current demand and clearing the backlog. Finally shortages will be cleared at time *t=T*. Then the cycle will be repeated indefinitely. The schematic diagram representing the inventory system is shown in Fig. 1

FIG. 1: SCHEMATIC DIAGRAM REPRESENTING THE INVENTORY LEVEL OF THE SYSTEM



The differential equations governing the system in the cycle time [0, T] are:

$$\frac{dI(t)}{dt}=\{\theta-\phi I(t)\}-\{\tau+\varphi I(t)\},\ 0\le t\le \gamma \quad (1)$$

$$\frac{dI(t)}{dt}=\{\theta-\phi I(t)\}-\{\alpha\beta(t-\gamma)^{\beta-1}I(t)\}-\{\tau+\varphi I(t)\},\ \gamma\le t\le t_1 \quad (2)$$

$$\frac{dI(t)}{dt}=-\{\alpha\beta(t-\gamma)^{\beta-1}I(t)\}-\{\tau+\varphi I(t)\},\ t_1\le t\le t_2 \quad (3)$$

$$\frac{dI(t)}{dt}=-\tau,\ t_2\le t\le t_3 \quad (4)$$

$$\frac{dI(t)}{dt}=\theta-\tau,\ t_3\le t\le T \quad (5)$$

with boundary conditions,

$$I(0)=0,\ I(t_2)=0,\quad I(T)=0$$

Solving the differential equations (1) to (5) the on-hand inventories at time *t* are respectively.

$$\mathrm{I}(t)=\frac{(\theta-\tau)}{\phi+\varphi}(1-e^{-(\phi+\varphi)t}),\quad 0\le t\le \gamma \quad (6)$$

$$\mathrm{I}(t)=(\theta-\tau)e^{-\left\{(\phi+\varphi)t+\alpha(t-\gamma)^{\beta}\right\}}\left\{\int_{\gamma}^{t}e^{(\phi+\varphi)u+\alpha(u-\gamma)^{\beta}}du+\frac{\left(e^{(\phi+\varphi)\gamma}-1\right)}{\phi+\varphi}\right\},\ \gamma\le t\le t_1 \quad (7)$$

$$I(t)=\tau e^{-\varphi t-\alpha(t-\gamma)^{\beta}}\int_{t}^{t_2}e^{\varphi u+\alpha(u-\gamma)^{\beta}}du,\ t_1\le t\le t_2 \quad (8)$$

$$I(t)=-\tau(t-t_2),\ t_2\le t\le t_3 \quad (9)$$

$$\mathrm{I}(t)=-(\theta-\tau)(T-t),\quad t_3\le t\le T \quad (10)$$

Since *I(t)* is continuous at t_1 evaluating equations (7) and (8) at $t=t_1$ and equating we get,

$$(\theta-\tau)e^{-\left\{(\phi+\varphi)t_1+\alpha(t_1-\gamma)^{\beta}\right\}}\left\{\int_{\gamma}^{t_1}e^{\left\{(\phi+\varphi)u+\alpha(u-\gamma)^{\beta}\right\}}du+\frac{\left(e^{(\phi+\varphi)\gamma}-1\right)}{\phi+\varphi}\right\}=\tau e^{-\varphi t-\alpha(t_1-\gamma)^{\beta}}\int_{t_1}^{t_2}e^{\varphi u+\alpha(u-\gamma)^{\beta}}du \quad (11)$$

This equality is used to establish the relationship between t_1 and t_2. Either side of equation (11) can be used as the value of the maximum inventory level, Q_1. Using Taylor's series expansion of the exponential function, ignoring terms of higher order and integrating we get,

$$Q_1=\tau e^{-\varphi t_1-\alpha(t_1-\gamma)^{\beta}}\left\{t_2+\frac{1}{2}\varphi t_2^{\,2}+\frac{\alpha(t_2-\gamma)^{\beta+1}}{(\beta+1)}-t_1-\frac{1}{2}\varphi t_1^{\,2}-\frac{\alpha(t_1-\gamma)^{\beta+1}}{(\beta+1)}\right\} \quad (12)$$

Similarly, since I(t) is continuous at t_3, evaluating equations (9) and (10) at $t=t_3$ and equating we get;

$$\tau(t_3-t_2)=(\theta-\tau)(T-t_3) \quad (13)$$

Therefore we have

$$t_3 = T - \frac{\tau(T - t_2)}{\theta} \tag{14}$$

Either side of equation (13) can be used as the value of the maximum shortage level, Q_2. Therefore,

$$Q_2 = -\frac{\tau}{\theta}(\theta - \tau)(T - t_2) \tag{15}$$

Backlogged demand is

$$B(t) = \int_{t_2}^{t_3} D(t)dt = \int_{t_2}^{t_3} (\tau - \varphi I(t))\, dt$$

$$= \tau(t_3 - t_2) - \frac{\varphi\tau}{2}(t_3 - t_2)^2 \tag{16}$$

Stock loss due to deterioration at any time *t* is

$$L(t) = \int_0^t R(t)dt - \int_0^t D(t)dt - I(t)$$

$$L(t) = \begin{cases} (\theta - \tau)t - (\phi + \varphi)\left\{\int_0^{\gamma} I(t)\,dt + \int_{\gamma}^{t} I(t)\,dt\right\} - I(t), & \gamma \le t \le t_1 \\ (\theta - \tau)t_1 - (\phi + \varphi)\left\{\int_0^{\gamma} I(t)\,dt + \int_{\gamma}^{t_1} I(t)\,dt\right\} - \tau(t - t_1) & \\ \quad -\varphi\int_{t_1}^{t} I(t)\,dt - I(t), & t_1 \le t \le t_2 \\ 0, & else\ where \end{cases} \tag{17}$$

where *I(t)* is as defined in equations (6) and (7).

Production quantity in the cycle time $(0, T)$ is

$$Q = \int_0^{\gamma} R(t)dt + \int_{\gamma}^{t_1} R(t)dt + \int_{t_3}^{T} R(t)dt$$

$$= \theta(t_1 + T - t_3) - \int_0^{\gamma} \phi I(t)\,dt - \int_{\gamma}^{t_1} \phi I(t)\,dt \tag{18}$$

Substituting *I(t)* in equation (18) by the expressions in equations (6) and (7), using Taylor's series expansion of the exponential function, ignoring terms of higher order and integrating we get

$$\begin{aligned} Q = {} & \theta t_1 + \tau(T - t_2) - \frac{\phi(\theta - \tau)}{(\phi + \varphi)^2}\left\{(\phi + \varphi)\gamma + e^{-(\phi+\varphi)\gamma} - 1\right\} \\ & -\phi(\theta - \tau)\Bigg\{ \mathrm{B}t_1 + \frac{1}{2}t_1^2(1 - (\phi + \varphi)B) - \frac{1}{6}(\phi + \varphi)t_1^3 - \frac{1}{8}(\phi + \varphi)^2 t_1^4 \\ & + \frac{2\alpha(t_1 - \gamma)^{\beta+2}}{(\beta+1)(\beta+2)} - \frac{\alpha(t_1 - \gamma)^{\beta+1}}{\beta+1}\left(B + t_1 + \frac{1}{2}(\phi + \varphi)t_1^2\right) - \frac{\alpha^2(t_1 - \gamma)^{2(\beta+1)}}{2(\beta+1)^2} - B\gamma \\ & -\frac{1}{2}\gamma^2(1 - (\phi + \varphi)B) + \frac{1}{6}(\phi + \varphi)\gamma^3 + \frac{1}{8}(\phi + \varphi)^2\gamma^4 \Bigg\} \end{aligned} \tag{19}$$

where, $\mathrm{B} = \frac{1}{\phi + \varphi}\left\{e^{(\phi+\varphi)\gamma} - 1\right\} - \frac{1}{2}(\phi + \varphi)\gamma^2 - \gamma$ (20)

The total cost per unit time of the system, $TC = TC(t_1, t_2)$, is the sum of setup cost per unit time, the production cost per unit time, inventory holding cost per unit time and the shortage cost per unit time, i.e.

$$TC(t_1,t_2)=\frac{A}{T}+\frac{pQ}{T}+\frac{h}{T}\left\{\int_0^{\gamma}I(t)\,dt+\int_{\gamma}^{t_1}I(t)\,dt+\int_{t_1}^{t_2}I(t)\,dt\right\}+\frac{c_2}{T}\left\{\int_{t_2}^{t_3}-I(t)\,dt+\int_{t_3}^{T}-I(t)\,dt\right\} \quad (21)$$

Substituting the expressions for I(t) from equations (6) to (10) in equation (21), using Taylor's series expansion of the exponential function, neglecting higher order terms and integrating we get,

$$TC=\frac{A}{T}+\frac{p\theta}{T}\left[t_1+\frac{\tau(T-t_2)}{\theta}\right]+\frac{h-p\phi}{T}\frac{(\theta-\tau)}{(\phi+\varphi)^2}\left\{(\phi+\varphi)\gamma+e^{-(\phi+\varphi)\gamma}-1\right\}$$

$$+\frac{(h-p\phi)}{T}(\theta-\tau)\Bigg\{Bt_1+\frac{1}{2}t_1^2(1-(\phi+\varphi)B)-\frac{1}{6}(\phi+\varphi)t_1^3-\frac{1}{8}(\phi+\varphi)^2t_1^4$$

$$+\frac{2\alpha(t_1-\gamma)^{\beta+2}}{(\beta+1)(\beta+2)}-\frac{\alpha(t_1-\gamma)^{\beta+1}}{\beta+1}\left(B+t_1+\frac{1}{2}(\phi+\varphi)t_1^2\right)-\frac{\alpha^2(t_1-\gamma)^{2(\beta+1)}}{2(\beta+1)^2}-B\gamma$$

$$-\frac{1}{2}\gamma^2(1-(\phi+\varphi)B)+\frac{1}{6}(\phi+\varphi)\gamma^3+\frac{1}{8}(\phi+\varphi)^2\gamma^4\Bigg\}$$

$$+\frac{\tau h}{T}\Bigg\{\left[t_2+\frac{1}{2}\varphi t_2^2+\frac{\alpha}{(\beta+1)}(t_2-\gamma)^{\beta+1}\right]\left[(t_2-t_1)-\frac{\varphi}{2}(t_2^2-t_1^2)-\frac{\alpha\{(t_2-\gamma)^{\beta+1}-(t_1-\gamma)^{\beta+1}\}}{(\beta+1)}\right]$$

$$-\frac{(t_2^2-t_1^2)}{2}+\frac{\varphi}{6}(t_2^3-t_1^3)+\frac{\varphi^2}{8}(t_2^4-t_1^4)+\frac{\alpha(t_2-\gamma)^{\beta+1}}{(\beta+1)}\left[t_2+\frac{1}{2}\varphi t_2^2\right]$$

$$-\frac{\alpha(t_1-\gamma)^{\beta+1}}{(\beta+1)}\left[t_1+\frac{1}{2}\varphi t_1^2\right]-\frac{2\alpha\left[(t_2-\gamma)^{\beta+2}-(t_1-\gamma)^{\beta+2}\right]}{(\beta+1)(\beta+2)}+\frac{\alpha^2\left[(t_2-\gamma)^{2(\beta+1)}-(t_1-\gamma)^{2(\beta+1)}\right]}{2(\beta+1)^2}\Bigg\}+\frac{c_2\tau}{2\theta T}(\theta-\tau)(T-t_2)^2 \quad (22)$$

3.2. OPTIMAL POLICIES OF THE MODEL

In this section, we obtain the optimal pricing and ordering policies of the inventory model developed in section (4.1). The total cost function is minimized to obtain the optimal production scheduling policies. To obtain these optimal values we differentiate *TC* in equation (22) with respect to t_1 and t_2 and equate the resulting equations to zero. And then the optimal values for the production up-time t_3, production quantity, *Q* and total cost, *TC* will be obtained using equations (14), (19) and (22) respectively.

The condition for the solutions to be optimal (minimum) is that the determinant of the Hessian matrix to be positive definite, i.e.

$$D=\begin{bmatrix}\frac{\partial^2TC(t_1,t_2)}{\partial t_1^2} & \frac{\partial^2TC(t_1,t_2)}{\partial t_1\partial t_2}\\ \frac{\partial^2TC(t_1,t_2)}{\partial t_1\partial t_2} & \frac{\partial^2TC(t_1,t_2)}{\partial t_2^2}\end{bmatrix}>0$$

Differentiating $TC(t_1,t_2)$ with respect to t_1 and equating to zero we get

$$\frac{p\theta}{T}+\frac{(h-p\phi)}{T}(\theta-\tau)\Bigg\{B+(1-(\phi+\varphi)B)t_1-\frac{1}{2}(\phi+\varphi)t_1^2-\frac{1}{2}(\phi+\varphi)^2t_1^3$$

$$+\frac{\alpha(t_1-\gamma)^{\beta+1}}{\beta+1}(1-(\phi+\varphi)t_1)-\alpha(t_1-\gamma)^{\beta}\left(B+t_1+\frac{1}{2}(\phi+\varphi)t_1^2\right)-\frac{\alpha^2(t_1-\gamma)^{2\beta+1}}{(\beta+1)}\Bigg\}$$

$$+\frac{\tau h}{T}\Bigg\{\left[t_2+\frac{1}{2}\varphi t_2^2\frac{\alpha}{(\beta+1)}(t_2-\gamma)^{\beta+1}\right]\left[\alpha(t_1-\gamma)^{\beta}+\varphi t_1-1\right]+t_1-\frac{1}{2}\varphi t_1^2-\frac{1}{2}\varphi^2t_1^3$$

$$-\alpha(t_1-\gamma)^{\beta}\left(t_1+\frac{1}{2}\varphi t_1^2\right)+\frac{\alpha(t_1-\gamma)^{\beta+1}}{(\beta+1)}(1-\varphi t_1)+\frac{\alpha^2(t_1-\gamma)^{2\beta+1}}{(\beta+1)}\Bigg\}=0 \quad (23)$$

Differentiating $TC(t_1,t_2)$ with respect to t_2 and equating to zero we get

$$-\frac{p\tau}{T}-\frac{\tau c_2}{\theta T}(\theta-\tau)(T-t_2)$$

$$+\frac{\tau h}{T}\left\{\left[1+\varphi t_2+\alpha(t_2-\gamma)^{\beta}\right]\left[(t_2-t_1)-\frac{\varphi}{2}(t_2^2-t_1^2)-\frac{\alpha\{(t_2-\gamma)^{\beta+1}-(t_1-\gamma)^{\beta+1}\}}{(\beta+1)}\right]\right.$$

$$+\left[t_2+\frac{1}{2}\varphi t_2^{\,2}+\frac{\alpha\,(t_2-\gamma)^{\beta+1}}{(\beta+1)}\right]\left[1-\varphi t_2-\alpha\,(t_2-\gamma)^{\beta}\right]-t_2+\frac{1}{2}\varphi t_2^{\,2}+\frac{1}{2}\varphi^2 t_2^{\,3}$$

$$+\alpha\,(t_2-\gamma)^{\beta}\left(t_2+\frac{1}{2}\varphi t_2^{\,2}\right)+\frac{\alpha\,(t_2-\gamma)^{\beta+1}(\varphi t_2-1)}{(\beta+1)}+\frac{\alpha^2\,(t_2-\gamma)^{2\beta+1}}{\beta+1}\Bigg\}=0 \qquad (24)$$

The solutions t_1^* and t_2^* of t_1 and t_2 are obtained by solving the nonlinear equations (23) and (24) by using numerical methods.

3.3. NUMERICAL ILLUSTRATION

Consider the case of deriving an EPQ and other optimal policies for a food processing industry for *T=6*. Here the product is of a deteriorating type and has a random lifetime which is assumed to follow a three parameter Weibull distribution. Records and discussions held with the production and marketing personnel suggest the values of various parameters. The deterioration parameters (α, β, γ) are estimated to vary from (0.03, 1, 0.4) to (0.06, 4, 0.7), stock dependent demand parameters from (50, 0.04) to (65, 0.07) and stock dependent production parameters from (0.4, 100) to (0.7, 130). Let the values for other parameters be *p*=6, *h*=4, c_2=2 and *A*=75 all in appropriate units. Substituting these values into the equations (23) and (24) and solving by using numerical methods we obtain the optimal solutions t_1^* and t_3^*, Q^* and TC^* of t_1 and t_3, *Q* and TC respectively. The values of the parameters are varied to observe the trend in the optimal policies and the results are presented in table 1.

TABLE 1: OPTIMAL SOLUTIONS OF THE MODEL WITH SHORTAGES FOR DIFFERENT VALUES OF THE PARAMETERS

	ф	ϕ	Θ	α	β	γ	*p*	*h*	c_2	*A*	t_1*	t_3*	Q*	TC*
50	0.05	0.60	120	0.04	2	0.50	6.0	4.0	2.0	75	2.635	5.668	421.499	453.064
55											2.596	5.550	419.652	466.593
60											2.542	5.420	416.741	480.281
65											2.474	5.274	414.221	494.152
60	0.04	0.60	120	0.04	2	0.50	6.0	4.0	2.0	75	2.667	5.460	442.054	484.621
	0.05										2.542	5.420	416.741	480.281
	0.06										2.417	5.377	393.974	475.969
	0.07										2.292	5.332	373.494	471.719
60	0.05	0.40	120	0.04	2	0.50	6.0	4.0	2.0	75	2.495	5.370	359.247	494.700
		0.50									2.436	5.313	374.375	483.127
		0.60									2.542	5.420	416.741	480.281
		0.70									2.681	5.751	497.700	492.262
60	0.05	0.60	100	0.04	2	0.50	6.0	4.0	2.0	75	2.356	5.042	344.390	430.317
			110								2.468	5.260	379.706	455.179
			120								2.542	5.420	416.741	480.281
			130								2.592	5.540	454.294	505.568
60	0.05	0.60	120	0.03	2	0.50	6.0	4.0	2.0	75	2.94	5.567	514.328	490.143
				0.04							2.542	5.420	416.741	480.281
				0.05							2.224	5.274	363.450	472.381
				0.06							1.977	5.149	333.909	466.306
60	0.05	0.60	120	0.04	1	0.50	6.0	4.0	2.0	75	3.677	5.747	813.967	506.786
					2						2.542	5.420	416.741	480.281
					3						1.549	4.838	308.194	463.598
					4						1.182	4.580	297.041	461.485
60	0.05	0.60	120	0.04	2	0.40	6.0	4.0	2.0	75	2.454	5.389	400.295	477.370
						0.50					2.542	5.420	416.741	480.281
						0.60					2.628	5.448	434.232	483.108
						0.70					2.711	5.474	452.409	485.839
60	0.05	0.60	120	0.04	2	0.50	5.0	4.0	2.0	75	2.143	4.982	380.975	413.471
							6.0				2.542	5.420	416.741	480.281
							7.0				2.522	5.764	370.295	546.873
							8.0				2.343	5.908	310.488	603.362
60	0.05	0.60	120	0.04	2	0.50	6.0	3.5	2.0	75	2.531	5.723	377.529	472.449
								4.0			2.542	5.420	416.741	480.281
								4.5			2.339	5.120	404.109	485.850
								5.0			2.161	4.924	391.322	492.145
60	0.05	0.60	120	0.04	2	0.50	6.0	4.0	1.0	75	2.679	5.471	448.429	477.215
									2.0		2.542	5.420	416.741	480.281
									3.0		2.358	5.348	381.010	484.059
									4.0		2.067	5.231	337.377	489.038
60	0.05	0.60	120	0.04	2	0.50	6.0	4.0	2.0	50	2.542	5.420	416.741	476.115
										75	2.542	5.420	416.741	480.281
										100	2.542	5.420	416.741	484.448

From table 1 it is observed that the demand rate parameters τ and ф have similar effects on the optimal values of the decision variables t_1^*, t_3^* and Q^*. An increase in these parameters decreases the values of the decision variables. Optimal values of the total cost per unit time TC^* will increase if τ is increased and decrease if ф is increased. The change in τ has relatively more significant effect on TC^* than that of the change in ф. The increase in the values of the production rate parameters ϕ and θ increase the values of all the decision variables t_1^*, t_3^*, Q^* and TC^*. Both the parameters have moderate to high influences on the optimal values.

The deterioration distribution parameters α and β have significant effect on t_1^* and Q^* and lower effects on t_3^* and TC^*. In all cases the increase in these parameters results in a decrease in the optimal values of the variables. When the unit production cost *p* increases, t_3^* also increases. On the contrary, t_3^* decreases when the unit holding cost *h* increases. Both parameters have increasing effects on TC^*. The unit shortage cost has negative influence on t_1^*, t_3^*, and Q^* and positive influence on TC^*.

3.4. SENSITIVITY ANALYSIS

To study the effects of changes in the parameters on the optimal values of the decision variables of the model we perform a sensitivity analysis. Let τ=60, ϕ=0.05, α=0.04, β=2, *γ*=0.5, θ=120, *φ*=0.6, *p*=6, *h*=4 and c_2=2 in appropriate units. Sensitivity analysis is then performed by changing these parameter values by -15%, -10%, -5%, 5%, 10% and 15%, first changing the value of one parameter at a time while keeping all the rest at their original values and then simultaneously changing the values of all the parameters. The results are presented in table 2. The relationships between parameters, costs and the optimal values are shown in Figure 2.

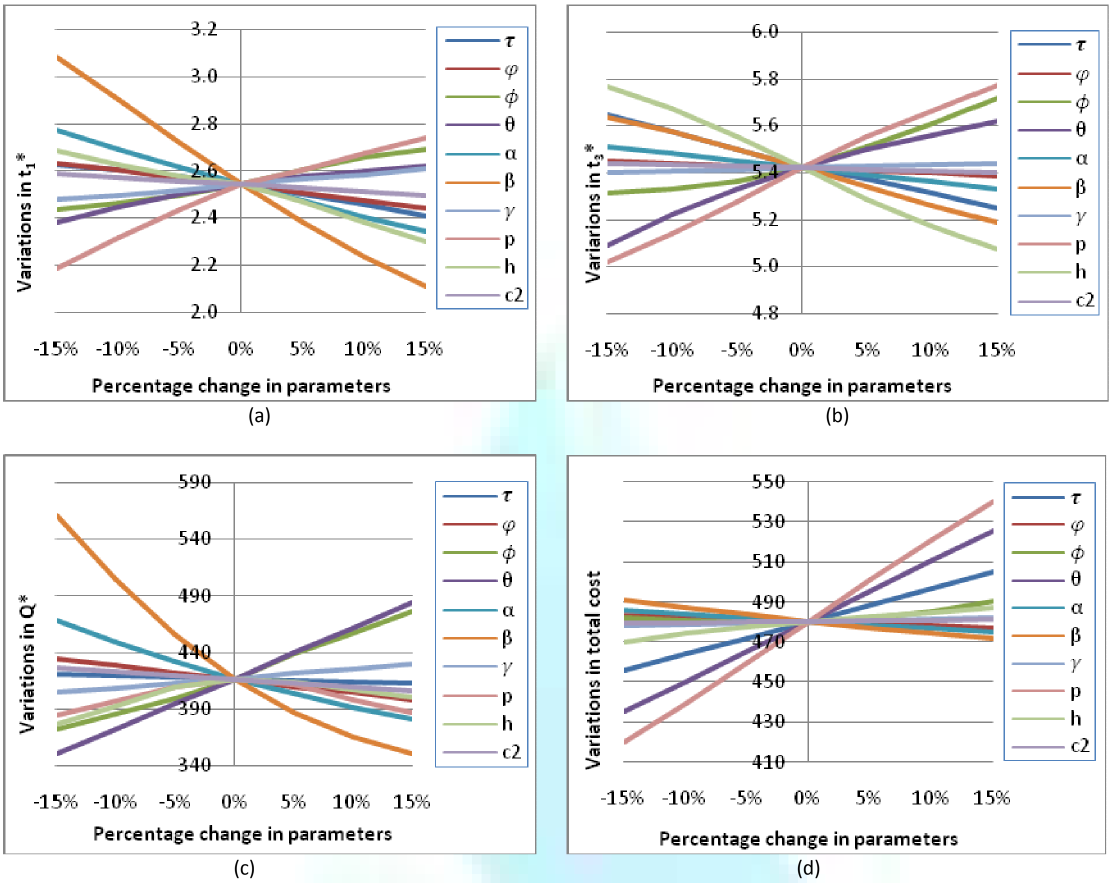
From table 2 it is observed that the production down-time t_1^* is highly sensitive to the changes in the deterioration distribution parameters α and β and the unit production cost *p*. For example a 15% decrease in β results in a 21.164% increase in t_1^* and a 15% increase in β to a 17.034% decrease in the optimal value of t_1^*. The demand and production rate parameters have moderately high influence on t_1^*. The production up-time t_3^* is moderately sensitive to the demand parameter τ, the production parameter θ and cost parameters *p* and *h*.

The production rate parameters θ and *φ* and the deterioration distribution parameters α and β have significant influences on the optimal values of the production quantity Q^*. A 15% decrease in α and β increases Q^* by 12.389% and 34.682% respectively and the same amount increase in θ results in a 16.150% increase in Q^*. The unit production cost and production parameter θ have significant influence on TC^*.

TABLE 2: SENSITIVITY ANALYSIS OF THE MODEL WITH SHORTAGES

arameter Values	Variable	Percentage Change in the parameter Values						
		-15%	**-10%**	**-5%**	**0%**	**+5%**	**+10%**	**+15%**
τ= 60	t_1*	2.628	2.605	2.576	2.542	2.503	2.459	2.410
	t_3*	5.646	5.574	5.499	5.420	5.334	5.344	5.147
	*Q**	421.11	420.162	418.894	416.741	415.136	413.844	413.149
	*TC**	455.759	463.875	472.047	480.281	488.582	496.949	505.382
ϕ=0.05	t_1*	2.630	2.605	2.567	2.542	2.504	2.479	2.442
	t_3*	5.447	5.439	5.428	5.420	5.407	5.399	5.386
	*Q**	434.361	429.252	421.624	416.741	409.513	404.908	398.311
	*TC**	483.319	482.450	481.148	480.281	478.983	478.119	476.827
φ=0.6	t_1*	2.437	2.454	2.488	2.542	2.606	2.660	2.681
	t_3*	5.315	5.329	5.362	5.420	5.503	5.609	5.717
	*Q**	376.695	386.085	398.872	416.741	438.922	461.310	476.469
	*TC**	482.408	480.766	479.992	480.281	481.928	485.226	490.242
θ=120	t_1*	2.382	2.449	2.501	2.542	2.575	2.600	2.620
	t_3*	5.092	5.222	5.329	5.420	5.496	5.561	5.618
	*Q**	351.135	372.420	394.372	416.741	439.470	461.786	484.043
	*TC**	435.267	450.185	465.194	480.281	495.435	510.642	525.893
α=0.04	t_1*	2.772	2.692	2.615	2.542	2.472	2.405	2.342
	t_3*	5.511	4.581	5.450	5.420	5.389	5.360	5.330
	*Q**	468.370	449.049	431.817	416.741	403.359	391.396	381.054
	*TC**	485.992	484.012	482.107	480.281	478.539	476.879	475.301
β=2	t_1*	3.080	2.903	2.719	2.542	2.380	2.236	2.109
	t_3*	5.635	5.574	5.500	5.420	5.338	5.261	5.189
	*Q**	561.274	504.868	455.751	416.741	387.446	366.006	350.260
	*TC**	491.021	487.384	483.733	480.281	477.166	474.441	472.107
γ=0.5	t_1*	2.481	2.498	2.520	2.542	2.564	2.585	2.607
	t_3*	5.398	5.404	5.412	5.420	5.426	5.434	5.441
	*Q**	405.244	408.362	412.481	416.741	421.420	425.296	429.860
	*TC**	478.251	478.836	479.561	480.281	480.996	481.706	482.410
p=6	t_1*	2.186	2.318	2.445	2.542	2.584	2.575	2.538
	t_3*	5.018	5.139	5.274	5.420	5.554	5.660	5.741
	*Q**	384.768	397.263	409.888	416.741	411.801	396.540	377.121
	*TC**	419.850	439.392	459.577	480.281	501.052	521.296	540.645
h=4	t_1*	2.500	2.555	2.574	2.542	2.469	2.382	2.299
	t_3*	5.767	5.672	5.554	5.420	5.287	5.171	5.074
	*Q**	364.293	389.847	408.992	416.741	414.264	407.653	401.000
	*TC**	469.943	474.572	477.849	480.281	482.444	484.676	487.060
c_2=2	t_1*	2.587	2.572	2.557	2.542	2.526	2.510	2.493
	t_3*	5.437	5.431	5.425	5.420	5.413	5.407	5.401
	*Q**	426.653	423.277	420.014	416.741	413.321	410.018	406.519
	*TC**	479.300	479.621	479.948	480.281	480.622	480.970	481.325
A=75	*TC**	478.417	479.031	479.656	480.281	480.906	481.531	482.156
All Parameters	t_1*	2.882	2.809	2.706	2.542	2.338	2.154	1.987
	t_3*	5.429	5.436	5.442	5.420	5.354	5.274	5.187
	*Q**	384.429	406.217	419.263	416.741	405.630	399.014	396.252
	*TC**	358.086	397.106	438.003	480.281	523.581	569.036	616.160

FIG. 2: SENSITIVITY ANALYSIS OF SYSTEM VARIABLES WITH RESPECT TO THE PARAMETERS AND COSTS FOR THE MODEL WITH SHORTAGES

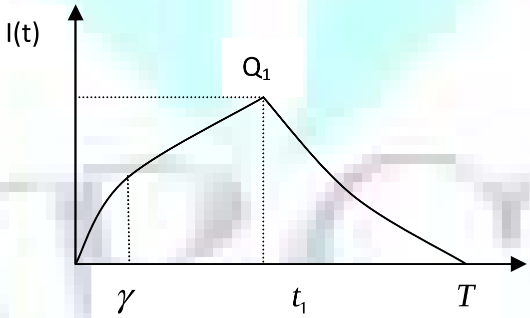


4. INVENTORY MODEL WITHOUT SHORTAGES

4.1. MODEL FORMULATION

Consider an inventory system for deteriorating items in which the lifetime of the commodity is random and follows three parameter Weibull distribution. Here it is assumed that shortages are not allowed. Then in this system the inventory level changes during $(0,\gamma)$ due to demand and production, during (γ,t_1) due to deterioration, demand and production and during (t_1,T) due to demand and deterioration. At time *T=0* the inventory is zero and production starts again. The schematic diagram representing the inventory system is shown in Fig. 3

FIG. 3: SCHEMATIC DIAGRAM REPRESENTING THE INVENTORY LEVEL OF THE SYSTEM WITH NO SHORTAGES



Given the assumptions in section (3.2) the differential equations governing the system in the cycle time [0, T] are:

$$\frac{dI(t)}{dt}=\{\theta-\phi I(t)\}-(\tau+\varphi I(t)),\ 0\le t\le\gamma \tag{25}$$

$$\frac{dI(t)}{dt}=\{\theta-\phi I(t)\}-\{\alpha\beta(t-\gamma)^{\beta-1}I(t)\}-(\tau+\varphi I(t),\ \ \gamma\le t\le t_1 \tag{26}$$

$$\frac{dI(t)}{dt}=-\{\alpha\beta(t-\gamma)^{\beta-1}I(t)\}-(\tau+\varphi I(t),\ t_1\le t\le T \tag{27}$$

With boundary conditions;

$I(0)=0$ and $I(T)=0$

Using the integrating factor $e^{(\phi+\varphi)t}$, the solutions of the differential equations (25) to (27) are respectively

$$I(t)=\frac{(\theta-\tau)}{\phi+\varphi}(1-e^{-(\phi+\varphi)t}),\ 0\le t\le\gamma \tag{28}$$

And

$$I(t)=(\theta-\tau)e^{-\left\{(\phi+\varphi)t+\alpha(t-\gamma)^{\beta}\right\}}\left\{B+t+\frac{(\phi+\varphi)}{2}t^2+\frac{\alpha(t-\gamma)^{\beta+1}}{\beta+1}\right\},\ \gamma\le t\le t_1 \tag{29}$$

where, *B* is as defined in (20)

Using similar procedure as that of the model with shortages we get the following results.

The maximum inventory level, Q_1 is:

$$Q_1=\tau e^{-\varphi t_1-\alpha(t_1-\gamma)^{\beta}}\left\{T-t_1+\frac{\varphi(T^2-t_1^2)}{2}+\frac{\alpha\left\{(T-\gamma)^{\beta+1}-(t_1-\gamma)^{\beta+1}\right\}}{(\beta+1)}\right\} \tag{30}$$

Stock loss due to deterioration at any time *t* is

$$L(t)=\int_0^t R(t)dt-\int_0^t D(t)dt-I(t)$$

$$L(t)=\begin{cases}(\theta-\tau)t-(\phi+\varphi)\left\{\int_0^{\gamma}I(t)dt+\int_{\gamma}^{t}I(t)dt\right\}-I(t),\ \gamma\le t\le t_1\\ (\theta-\tau)t_1-(\phi+\varphi)\left\{\int_0^{\gamma}I(t)dt+\int_{\gamma}^{t_1}I(t)dt\right\}-\tau(t-t_1)\\ \qquad -\varphi\int_{t_1}^{t}I(t)dt-I(t),\qquad t_1\le t\le T\\ 0,\qquad else\ where\end{cases} \tag{31}$$

Total Production in the cycle time $(0,T)$ is

$$Q=\int_0^{\gamma}R(t)dt+\int_{\gamma}^{t_1}R(t)dt$$

This implies

$$\begin{aligned}Q=\theta t_1-\frac{\phi(\theta-\tau)}{(\phi+\varphi)^2}\left\{(\phi+\varphi)\gamma+e^{-(\phi+\varphi)\gamma}-1\right\}\\ -\phi(\theta-\tau)\Bigg\{Bt_1+\frac{1}{2}t_1^2(1-(\phi+\varphi)B)-\frac{1}{6}(\phi+\varphi)t_1^3-\frac{1}{8}(\phi+\varphi)^2t_1^4\\ +\frac{2\alpha(t_1-\gamma)^{\beta+2}}{(\beta+1)(\beta+2)}-\frac{\alpha(t_1-\gamma)^{\beta+1}}{\beta+1}\left(B+t_1+\frac{1}{2}(\phi+\varphi)t_1^2\right)-\frac{\alpha^2(t_1-\gamma)^{2(\beta+1)}}{2(\beta+1)^2}\\ -B\gamma-\frac{1}{2}\gamma^2(1-(\phi+\varphi)B)+\frac{1}{6}(\phi+\varphi)\gamma^3+\frac{1}{8}(\phi+\varphi)^2\gamma^4\Bigg\}\end{aligned} \tag{32}$$

Total cost is the sum of setup cost per unit time, the production cost per unit time and inventory holding cost per unit time. In this model the cycle time is also assumed to be a decision variable. Let $TC(t_1,T)=TC$ be the total cost per unit time for this model.

Therefore

$$TC=\frac{A}{T}+\frac{pQ}{T}+\frac{h}{T}\left\{\int_0^{\gamma}I(t)dt+\int_{\gamma}^{t_1}I(t)dt+\int_{t_1}^{T}I(t)dt\right\}$$

This implies

$$TC=\frac{A}{T}+\frac{p\theta t_1}{T}+\frac{h-p\phi}{T}\frac{(\theta-\tau)}{(\phi+\varphi)^2}\left\{(\phi+\varphi)\gamma+e^{-(\phi+\varphi)\gamma}-1\right\}$$

$$+\frac{(h-p\phi)}{T}(\theta-\tau)\left\{Bt_1+\frac{1}{2}t_1^2(1-(\phi+\varphi)B)-\frac{1}{6}(\phi+\varphi)t_1^3-\frac{1}{8}(\phi+\varphi)^2t_1^4\right.$$

$$+\frac{2\alpha(t_1-\gamma)^{\beta+2}}{(\beta+1)(\beta+2)}-\frac{\alpha(t_1-\gamma)^{\beta+1}}{\beta+1}\left(B+t_1+\frac{1}{2}(\phi+\varphi)t_1^2\right)-\frac{\alpha^2(t_1-\gamma)^{2(\beta+1)}}{2(\beta+1)^2}-B\gamma$$

$$\left.-\frac{1}{2}\gamma^2(1-(\phi+\varphi)B)+\frac{1}{6}(\phi+\varphi)\gamma^3+\frac{1}{8}(\phi+\varphi)^2\gamma^4\right\}$$

$$+\frac{h}{T}\tau\left\{\left[T+\frac{1}{2}\varphi T^2+\frac{\alpha(T-\gamma)^{\beta+1}}{(\beta+1)}\right]\left[(T-t_1)-\frac{\varphi}{2}(T^2-t_1^2)-\frac{\alpha\{(T-\gamma)^{\beta+1}-(t_1-\gamma)^{\beta+1}\}}{(\beta+1)}\right]\right.$$

$$-\frac{1}{2}(T^2-t_1^2)+\frac{\varphi}{6}(T^3-t_1^3)+\frac{\varphi^2}{8}(T^4-t_1^4)+\frac{\alpha(T-\gamma)^{\beta+1}}{(\beta+1)}\left[T+\frac{1}{2}\varphi T^2\right]$$

$$-\frac{\alpha(t_1-\gamma)^{\beta+1}}{(\beta+1)}\left[t_1+\frac{1}{2}\varphi t_1^2\right]-\frac{2\alpha\left[(T-\gamma)^{\beta+2}-(t_1-\gamma)^{\beta+2}\right]}{(\beta+1)(\beta+2)}+\frac{\alpha^2\left[(T-\gamma)^{2(\beta+1)}-(t_1-\gamma)^{2(\beta+1)}\right]}{2(\beta+1)^2}$$

(33)

4.2. Optimal policies of the model

The problem here is to find out the optimal values of the production down-time, t_1 and cycle length, T that minimize the total cost over the interval $[0,T]$. To obtain these values we differentiate $TC(t_1,T)$ in equation (33) with respect to t_1 and T and equate them to zero.
The condition for the solutions to be optimal (minimum) is that the determinant of the Hessian matrix to be positive definite, i.e.

$$D=\begin{bmatrix}\frac{\partial^2 TC(t_1,T)}{\partial t_1^2} & \frac{\partial^2 TC(t_1,T)}{\partial t_1\partial T}\\ \frac{\partial^2 TC(t_1,T)}{\partial t_1\partial T} & \frac{\partial^2 TC(t_1,T)}{\partial T^2}\end{bmatrix}>0$$

Differentiating $TC(t_1,T)$ with respect to t_1 and equating to zero we get

$$\frac{p\theta}{T}+\frac{(h-p\phi)}{T}(\theta-\tau)\left\{B+(1-(\phi+\varphi)B)t_1-\frac{1}{2}(\phi+\varphi)t_1^2-\frac{1}{2}(\phi+\varphi)^2t_1^3\right\}$$

$$+\frac{\alpha}{\beta+1}(t_1-\gamma)^{\beta+1}(1-(\phi+\varphi)t_1)-\alpha(t_1-\gamma)^{\beta}\left(B+t_1+\frac{1}{2}(\phi+\varphi)t_1^2\right)$$

$$-\frac{\alpha^2(t_1-\gamma)^{2\beta+1}}{(\beta+1)}+\frac{\tau h}{T}\left\{\left[T+\frac{1}{2}\varphi T^2\frac{\alpha}{(\beta+1)}(T-\gamma)^{\beta+1}\right]\left[\alpha(t_1-\gamma)^{\beta}+\varphi t_1-1\right]+t_1\right.$$

$$\left.-\frac{1}{2}\varphi t_1^2-\frac{1}{2}\varphi^2t_1^3-\alpha(t_1-\gamma)^{\beta}\left(t_1+\frac{1}{2}\varphi t_1^2\right)+\frac{\alpha(t_1-\gamma)^{\beta+1}}{(\beta+1)}(1-\varphi t_1)-\frac{\alpha^2(t_1-\gamma)^{2\beta+1}}{(\beta+1)}\right\}=0 \quad (34)$$

Differentiating $TC(t_1,T)$ with respect to T and equating to zero we get

$$-\frac{A}{T^2}-\frac{p\theta t_1}{T^2}-\frac{h-p\phi}{T^2}\frac{(\theta-\tau)}{(\phi+\varphi)^2}\left\{e^{-\gamma(\phi+\varphi)}+\gamma(\phi+\varphi)-1\right\}$$

$$-\frac{(h-p\phi)}{T^2}(\theta-\tau)\left\{Bt_1+\frac{1}{2}t_1^2(1-(\phi+\varphi)B)-\frac{1}{6}(\phi+\varphi)t_1^3-\frac{1}{8}(\phi+\varphi)^2t_1^4\right.$$

$$+\frac{2\alpha(t_1-\gamma)^{\beta+2}}{(\beta+1)(\beta+2)}-\frac{\alpha(t_1-\gamma)^{\beta+1}}{\beta+1}\left(B+t_1+\frac{1}{2}(\phi+\varphi)t_1^2\right)-\frac{\alpha^2(t_1-\gamma)^{2(\beta+1)}}{2(\beta+1)^2}$$

$$-B\gamma-\frac{1}{2}\gamma^2\left(1-(\phi+\varphi)B\right)+\frac{1}{6}(\phi+\varphi)\gamma^3+\frac{1}{8}(\phi+\varphi)^2\gamma^4\Big\}$$

$$+\tau h\left\{\left[\frac{1}{2}\varphi+\frac{\alpha}{T}(T-\gamma)^{\beta}-\frac{\alpha(T-\gamma)^{\beta+1}}{T^2(\beta+1)}\right]\left[(T-t_1)-\frac{\varphi}{2}\left(T^2-t_1^{\,2}\right)-\frac{\alpha\left\{(T-\gamma)^{\beta+1}-(t_1-\gamma)^{\beta+1}\right\}}{(\beta+1)}\right]\right.$$

$$+\left[1+\frac{1}{2}\varphi T+\frac{\alpha(T-\gamma)^{\beta+1}}{T(\beta+1)}\right]\left[1-\varphi T-\alpha(T-\gamma)^{\beta}\right]-\frac{1}{2}-\frac{t_1^{\,2}}{2T^2}\left(\frac{\varphi^2 t_1^{\,2}}{4}+\frac{\varphi t_1}{3}-1\right)$$

$$+\frac{\varphi}{3}T+\frac{3}{8}\varphi^2T^2+\frac{\alpha(T-\gamma)^{\beta+1}}{(\beta+1)}\left(\frac{\varphi}{2}-\frac{2}{T}\right)+\alpha(T-\gamma)^{\beta+1}\left(1+\frac{1}{2}\varphi T\right)$$

$$+\frac{\alpha(t_1-\gamma)^{\beta+1}}{T^2(\beta+1)}\left[t_1+\frac{1}{2}\varphi t_1^{\,2}\right]+\frac{2\alpha\left[(T-\gamma)^{\beta+2}-(t_1-\gamma)^{\beta+2}\right]}{T^2(\beta+1)(\beta+2)}$$

$$\left.-\frac{\alpha^2\left[(T-\gamma)^{2(\beta+1)}-(t_1-\gamma)^{2(\beta+1)}\right]}{2T^2(\beta+1)^2}+\frac{\alpha^2(T-\gamma)^{2\beta+1}}{(\beta+1)T}\right\}=0 \qquad (35)$$

4.3. NUMERICAL ILLUSTRATION

Consider the case of deriving an economic production quantity and other optimal policies for a food processing industry considered in example 1. The optimal values t_1^*, T^*, Q^* and TC^* of t_1, T, Q and TC for different values of costs and parameters are determined and presented in table 3.

TABLE 3: OPTIMAL SOLUTIONS OF THE MODEL WITH SHORTAGES FOR DIFFERENT VALUES OF THE PARAMETERS

τ	ɸ	ϕ	θ	α	β	γ	p	h	A	t_1*	T*	Q*	TC*
50	0.05	0.6	120	0.04	2	0.50	5.0	4.0	75	1.462	4.346	155.427	467.681
55										1.589	4.242	172.491	482.429
60										1.691	4.150	187.238	495.681
65										1.774	4.069	199.911	507.432
60	0.04	0.6	120	0.04	2	0.50	5.0	4.0	75	1.784	4.233	199.218	500.801
	0.05									1.691	4.150	187.238	495.681
	0.06									1.604	4.072	176.446	490.577
	0.07									1.525	3.997	166.970	485.457
60	0.05	0.4	120	0.04	2	0.50	5.0	4.0	75	1.624	4.242	175.812	514.581
		0.5								1.668	4.167	180.970	502.880
		0.6								1.691	4.150	187.238	495.681
		0.7								1.690	4.208	193.962	494.345
60	0.05	0.6	100	0.04	2	0.50	5.0	4.0	75	1.855	3.961	177.483	437.868
			110							1.775	4.058	183.479	467.507
			120							1.691	4.150	187.238	495.681
			130							1.601	4.238	188.647	522.448
60	0.05	0.6	120	0.03	2	0.50	5.0	4.0	75	2.069	4.521	244.124	507.312
				0.04						1.691	4.150	187.238	495.681
				0.05						1.434	3.878	155.138	485.189
				0.06						1.247	3.669	133.960	475.963
60	0.05	0.6	120	0.04	1	0.50	5.0	4.0	75	3.533	5.482	705.207	519.143
					2					1.691	4.150	187.238	495.681
					3					0.942	3.294	101.602	468.466
					4					0.603	2.901	66.743	453.539
60	0.05	0.6	120	0.04	2	0.40	5.0	4.0	75	1.601	4.067	175.879	491.502
						0.50				1.691	4.150	187.238	495.681
						0.60				1.781	4.232	199.080	499.600
						0.70				1.871	4.312	211.464	503.245
60	0.05	0.6	120	0.04	2	0.50	5.0	4.0	75	1.810	3.826	203.915	439.486
							6.0			1.691	4.150	187.238	495.681
							7.0			1.516	4.334	164.797	543.491
							8.0			1.347	4.460	144.944	584.531
60	0.05	0.6	120	0.04	2	0.50	5.0	3.5	75	1.536	4.308	167.254	472.003
								4.0		1.691	4.150	187.238	495.681
								4.5		1.793	3.961	201.452	514.231
								5.0		1.830	3.746	206.850	529.415
60	0.05	0.6	120	0.04	2	0.50	5.0	4.0	50	1.707	4.164	189.407	490.077
									75	1.691	4.150	187.238	495.681
									100	1.674	4.136	184.957	501.313
									125	1.656	4.122	182.567	506.974

From table 3 it is observed that the optimal production down-time t_1^*, increases when the demand parameter τ, the production rate parameter ϕ, the deterioration distribution parameter γ, and the unit holding cost h are increasing. On the other hand the optimal cycle length T^* increases when the values of θ, γ and p increase and decrease when other parameters increase. The optimal production quantity Q^* increases when the values of τ, ϕ, θ, γ and h are increasing, and decreases when the values of other parameters are increase. The increase in τ, θ, γ and all cost parameters results in an increase in TC^*.

4.4. Sensitivity Analysis

To study the effects of changes in the parameters on the optimal values of the decision variables of the model we perform a sensitivity analysis. The sensitivity analysis of the model without shortages is carried with the same set of parameters as that of the model with shortages and the results are presented in table 4. The relationships between production quantity and parameters and that of optimal total cost and parameters are shown in Figures 5 and 6 respectively.

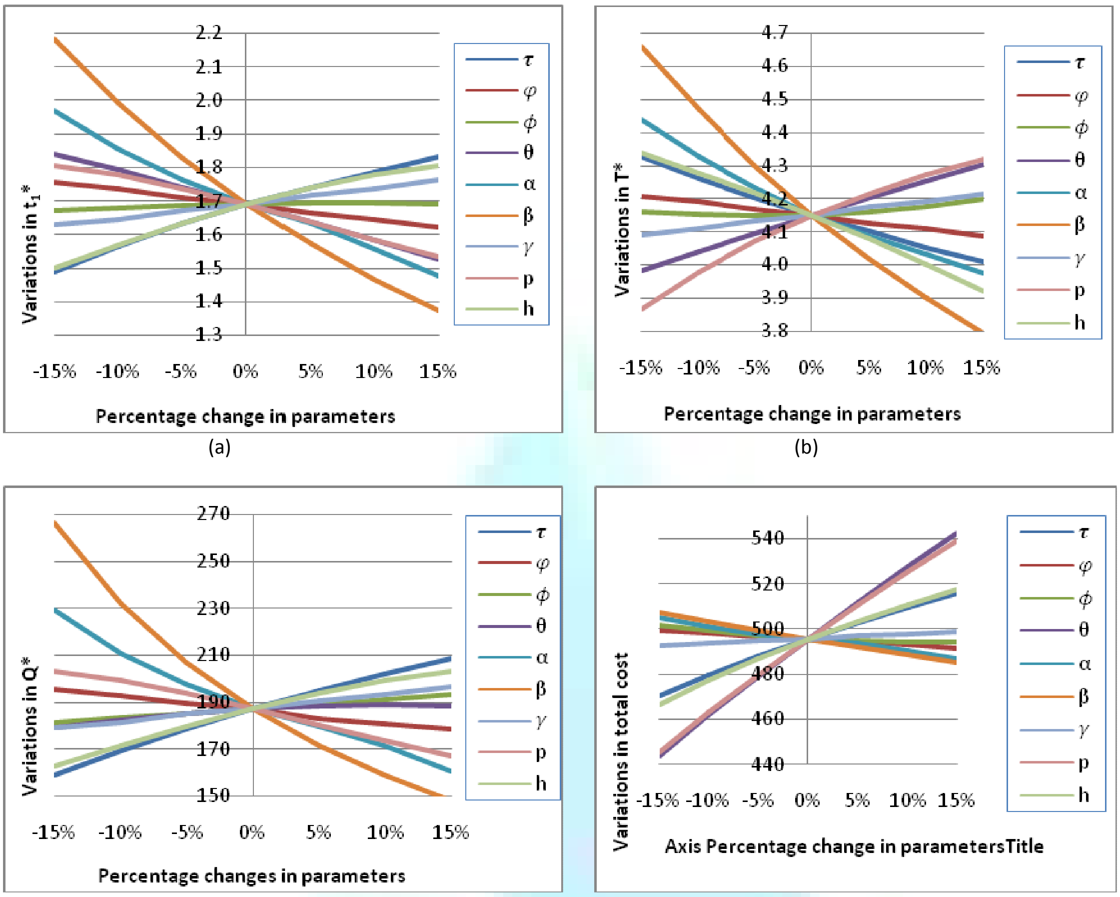
It can be observed from table 4 that the demand parameter τ has significant influence on the optimal values of t_1^* and Q^* and the production parameter θ a moderately high effect on the optimal values t_1^* and TC^*. The deterioration distribution parameters α and β immensely influence the values of t_1^* and Q^*. For example a 15% decrease in α increases t_1^* by 17.031% and Q^* by 22.450%. The same amount of decrease in β increases t_1^* by 29.154%, T^* by 12.265% and Q^* by 42.127%. The increase in these parameter values also have a significant decreasing effect on t_1^*, T^* and Q^*.The location parameter γ has a relatively lower effect on the decision variables. The cost parameters p and h also have significant influence on t_1^* and Q^*.

The optimal TC^* is highly sensitive to the changes in θ and p and slightly sensitive to others. Generally the optimal production down-time and the production quantity are more sensitive to the changes in the parameter values than the optimal cycle time and total cost.

TABLE 4: SENSITIVITY ANALYSIS OF THE MODEL WITH SHORTAGES

Parameter Values	Variable	Percentage Change in the parameter Values						
		-15%	**-10%**	**-5%**	**0%**	**+5%**	**+10%**	**+15%**
τ=60	t_1*	1.490	1.566	1.632	1.691	1.742	1.788	1.830
	T*	4.325	4.262	4.204	4.150	4.101	4.054	4.011
	Q*	159.067	169.294	178.599	187.238	194.969	202.116	208.762
	TC*	470.494	479.570	487.940	495.681	502.914	509.630	515.948
ϕ=0.05	t_1*	1.755	1.736	1.709	1.691	1.664	1.647	1.621
	T*	4.208	4.191	4.167	4.150	4.127	4.111	4.087
	Q*	195.425	192.967	189.518	187.238	181.198	181.735	178.526
	TC*	499.277	498.245	496.728	495.681	494.189	493.143	491.589
ϕ=0.6	t_1*	1.671	1.679	1.686	1.691	1.693	1.694	1.692
	T*	4.162	4.152	4.148	4.150	4.160	4.177	4.199
	Q*	181.514	183.246	185.196	187.238	189.217	191.386	193.426
	TC*	501.924	499.380	497.291	495.681	494.633	494.147	494.191
θ=120	t_1*	1.840	1.792	1.742	1.691	1.637	1.583	1.527
	T*	3.981	4.039	4.096	4.150	4.203	4.255	4.304
	Q*	178.952	182.551	185.257	187.238	188.257	188.759	188.503
	TC*	443.919	461.698	478.966	495.681	511.899	527.631	542.810
α=0.04	t_1*	1.979	1.823	1.754	1.691	1.632	1.577	1.479
	T*	4.437	4.285	4.215	4.150	4.089	4.032	3.926
	Q*	229.273	205.565	195.804	187.238	179.489	172.483	160.473
	TC*	504.945	500.261	497.940	495.681	493.474	491.334	487.165
β =2	t_1*	2.184	1.992	1.829	1.691	1.571	1.467	1.376
	T*	4.659	4.470	4.301	4.150	4.018	3.900	3.795
	Q*	266.115	232.110	206.697	187.238	171.595	158.840	148.192
	TC*	507.169	503.395	499.520	495.681	492.056	488.610	485.393
γ=0.5	t_1*	1.628	1.646	1.673	1.691	1.718	1.736	1.763
	T*	4.092	4.109	4.134	4.150	4.175	4.191	4.216
	Q*	179.239	181.502	184.929	187.238	190.737	193.095	196.670
	TC*	492.783	493.636	494.882	495.681	496.892	497.669	498.845
p=6	t_1*	1.805	1.778	1.738	1.691	1.639	1.586	1.534
	T*	3.867	3.979	4.073	4.150	4.216	4.271	4.319
	Q*	203.188	199.302	193.672	187.238	180.333	173.510	167.007
	TC*	445.436	462.925	479.714	495.681	510.898	525.316	539.046
h=4	t_1*	1.500	1.570	1.634	1.691	1.739	1.778	1.806
	T*	4.337	4.278	4.217	4.150	4.079	4.001	3.919
	Q*	162.850	171.489	179.681	187.238	193.811	199.302	203.333
	TC*	466.558	477.558	486.914	495.681	503.651	510.847	517.460
A=75	t_1*	1.698	1.696	1.693	1.691	1.688	1.685	1.683
	T*	4.157	4.155	4.153	4.150	4.148	4.146	4.144
	Q*	188.184	187.913	187.508	187.238	186.834	186.430	186.161
	TC*	493.188	494.020	494.877	495.681	496.539	497.373	498.220
All Parameters	t_1*	2.454	2.162	1.893	1.691	1.525	1.401	1.296
	T*	4.763	4.580	4.354	4.150	3.966	3.817	3.686
	Q*	250.179	225.697	202.707	187.238	175.157	167.285	160.862
	TC*	375.471	415.177	454.852	495.681	537.272	581.049	626.090

FIGURE 4: SENSITIVITY ANALYSIS OF SYSTEM VARIABLES WITH RESPECT TO THE PARAMETERS AND COSTS FOR THE MODEL WITHOUT SHORTAGES.



(a) (b) (c) (d)

5. CONCLUSION

In this paper, a production inventory model for deteriorating items with stock dependent demand and Weibull decay for both with and without shortages has been developed and analyzed in the light of various parameters and cost. The optimal production schedule is derived. This model also includes the exponential decay model as a particular case for specific values of the parameters. This model is having potential applications in manufacturing and production industries like Cement and Food processing, where the deterioration of the commodity is random and follows Weibull distribution. It is also observed that in the industries the demand is a function of stock on hand. The proposed models can further be enriched by incorporating salvage of deteriorated units, inflation, quantity discount, and trade credits etc. It can also be extended to a multi-commodity model with constraints on budget, shelf space, etc., These models may also be formulated in fuzzy environments.

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