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   OBJECTIVES

   HYPOTHESES

   RESEARCH METHODOLOGY

   RESULTS & DISCUSSION

   FINDINGS

   RECOMMENDATIONS/SUGGESTIONS

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   APPENDIX/ANNEXURE

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CONTRIBUTIONS TO BOOKS

- Sharma T., Kwatra, G. (2008) Effectiveness of Social Advertising: A Study of Selected Campaigns, Corporate Social Responsibility, Edited by David Crowther & Nicholas Capaldi, Ashgate Research Companion to Corporate Social Responsibility, Chapter 15, pp 287-303.

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- Schemenner, R.W., Huber, J.C. and Cook, R.L. (1987), "Geographic Differences and the Location of New Manufacturing Facilities," Journal of Urban Economics, Vol. 21, No. 1, pp. 83-104.

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A FUZZY EOQ INVENTORY MODEL WITH LEARNING EFFECTS INCORPORATING RAMP –TYPE DEMAND, PARTIAL BACKLOGGING AND INFLATION UNDER TRADE CREDIT FINANCING

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ABSTRACT

An EOQ Inventory model for a deteriorating item with ramp – type demand is developed in fuzzy stochastic environment with random Weibull distribution under inflation and time value of money over a finite planning horizon, when delay in payment is allowed to the retailer to settle the accounts against the purchases made by him. Here, we have considered two cases: (1) payment within the permissible time and (2) payment after the permissible time. Shortages are allowed. Here, we propose a mathematical model and theorem to find minimum total relevant inventory cost and optimal order quantity. Firstly, we consider the demand and net inflation rate to be crisp in nature. The holding, purchasing, shortage, lost, selling, and ordering costs are represented by triangular fuzzy numbers which are then transformed to corresponding weighted interval numbers by using nearest interval approximation. Following interval mathematics, the single objective fuzzy problem is reduced to a crisp multi – objective decision making (MODM) problems. The MODM problem is again transformed to a crisp single objective problem with the help of weighted sum method. The demand rate and the net inflation rate are taken as trapezoidal fuzzy numbers to make the problem much more realistic and derive the expressions for the total inventory cost applying Function Principle and then defuzzified using graded mean integration representation method. Numerical examples are cited to illustrate the developed mode. Some sensitivity analysis is carried out. We have applied the learning effects to further improve the optimal order quantity.

KEYWORDS
Trade Credit Financing, random Weibull distribution, fuzzy ramp-type demand, function principle, learning effects.

INTRODUCTION

In the inventory model, deterioration takes a substantial role for analysis of required results. It can be found in the form of decay, change, damage, spoilage or obsolescence that results in decreasing usefulness from its original purpose. Some kinds of production like food items (vegetables, fruits, milk, etc.), drugs, pharmaceuticals and radioactive substances are few examples in which appreciable deterioration can take place during the normal storage period of the units and consequently this loss must be taken into account when analyzing the model. Hence, many authors have considered Economic Order Quantity (EOQ) inventory models for deteriorating items with exponential decay proportional to the on-hand inventory. Hwang [1997] proposed a model in which inventory deteriorates with time. Deng [2005], Tripathy et. al. [2010], Chang et. al. [2010] considered the market demand of the item to be constant or time-dependent in their works. But in the real market demand of the product is always in dynamic state due to variability of time, price and stock displayed in inventory level. This impressed researchers and marketing practitioners to think about ramp type demand which increases with time upto a certain limit and then ultimately stabilizes and becomes constant. Such type of demand is observed in the items as newly launched mobile phones, fashion goods, garments, cosmetics etc.

Mondal et. al. [1998] presented an order level inventory model for deteriorating items with ramp type demand. Wu and Ouyang [2000] extended their model by the concept of shortages. Giri et al. [2003] and Wu et al. [1999] developed an EOQ model with Weibull deterioration, shortages and ramp type demand. Peter Shaohua Dang [2005] further generalized the Wu et al.[1999] and Jain and Kumar [2007] further generalized Wu and Ouyang [2000] model by allowing Weibull deterioration along with some proposed theorems to find the time at which on-hand inventory reaches to zero. Panda et al. [2008] gave an optimal replenishment policy for perishable seasonal product with ramp type demand rate. Sharma et al. [2009] developed an EOQ model for variable rate of deterioration having a ramp type demand rate. Kawaktsu [2010] presented a paper with ramp type demand and finite planning horizon. Pathak et al. [2010] developed a model with Weibull deterioration and shortages. Chang et al. [2010] considered a partial backlogging, inflation in their model. Pathak et al. [2010] developed a model for three plants with time-dependent fuzzy inflation and inflation-dependent demand by using interval arithmetic and random deterioration with partial backlogging and learning effects. Chen [1985] used function principle for operations on fuzzy numbers and Chen et. al. [1998] proposed a paper for graded mean integration representation of generalized fuzzy numbers. Mahata et. al. [2010] and Chen et al. [2005] presented a model by using graded mean integration representation and function principle.

An EOQ model with permissible delay in payments was developed by Goyal [1985] where he did not consider the difference between the selling price and purchasing cost. Goyal's model was improved by Dave [1985] under the assumption that the selling price is higher than the purchase price. Inventory models for optimal pricing and ordering policies for the retailers with trade credit were formulated by Whang et.al. [1997] and Liao et al. [2000]. Considering the difference between unit sell price and unit purchase cost, Jamal et al. [1997 and 2000] and Sarkar et al. [2000] suggested that the retailer should settle the account as soon as the unit selling price increases relative to the unit cost. Chang et al. [2003] have suggested a model under trade credit if the order quantity is greater than or equal to pre-determined quantity. Ouyang et. al. [2006], Chang et al. [2006], Chung and Huang [2009] and Teng et al.[2005] have suggested the strategy of granting credit items by adding not only an additional cost but also default risk to the supplier. Ouyang et al. [2009] have considered trade credit linked to order quantity for deteriorating items. More discussions are given in notes by Mitra et al. [1980], Giri et al. [2000] and Khanna et.al. [2005]. Shah et. al. [2010] presented their model with delayed in payment.

In this paper, an EOQ Inventory model for a deteriorating item with ramp – type demand is developed in fuzzy stochastic environment under inflation and time value of money over a finite planning horizon with single cycle, when delay in payment is allowed to the retailer to settle the accounts against the purchases made by him. Here, the case of the retailers generating revenue on unit selling price, higher than the unit purchase cost, has been considered. In this paper we have considered two cases: (1) payment within the permissible time and (2) payment after the permissible time (i.e. time at which on hand inventory reaches to zero). Here, shortages are allowed and the deterioration follows random Weibull distribution. Under these assumptions, we propose a mathematical model and theorem to find minimum total relevant inventory cost and optimal order quantity. Firstly, we consider the demand and net inflation rate to be crisp in nature. The holding, purchasing, shortage, lost, selling, and ordering costs are represented by triangular fuzzy numbers which are then transformed to corresponding weighted interval numbers by using nearest interval approximation. Following interval mathematics, the single objective fuzzy problem is reduced to a crisp multi – objective decision making (MODM) problems. The MODM problem for optimizing the total relevant inventory cost is then again transformed to a crisp single objective problem with the help of weighted sum method. Next, the demand rate and the net inflation rate are taken as trapezoidal fuzzy numbers to make the problem much more realistic and then derive the expressions for the total inventory cost applying Function Principle. It is then defuzzified using graded mean integration representation method to find the possibilistic mean value of the objective i.e. to get the total cost function. Numerical examples are cited to illustrate the developed model and the solution process. Some sensitivity analysis with respect to critical parameters is carried out to observe the changes in the total relevant inventory cost and optimal order quantity. Analyzing these changes, we have applied the learning effects to further improve the optimal order quantity. The percentage of defective rate is reduced with learning effect and due to this reduction the order quantity increases in every consecutive planning horizon. This increment in order quantity follows S-shape learning curve and after a certain number of consecutive cycle, it becomes constant. Finally the result of the objective for crisp demand and crisp net inflation rate with and without learning effects are compared to that for fuzzy demand and fuzzy net inflation rate in both case (1) and case (2).

ASSUMPTIONS AND NOTATIONS

This mathematical model is developed on the basis of the following assumptions and notations:

ASSUMPTIONS

1. Only a single-product item is considered during the planning horizon.
2. Replenishment rate is infinite; thus, replenishment is instanteneous.
3. A Discounted Cash Flow approach is used to consider the various costs at various times.
4. The time horizon is finite with single cycle. Lead time is negligible.
5. Shortages are allowed and partially backlogged. The backlogging rate is a decreasing function of the waiting time. Let the backlogging rate be B (T- t) $= e^{-\delta(T-t)}$, where δ ≥ 0, and (T-t) is the waiting time up to the next replenishment.
6. The random deterioration rate function , $\ddot{\theta}$ (t), represents the on-hand inventory deteriorates per unit time and there is no replacement or repair of deteriorated units during the period T and it satisfies two-parameter Weibull distribution. In the present model, we assume $\ddot{\theta}$ (t) = $\hat{\alpha}\,\beta t^{\beta-1}$,β > 1, 0 ≤ t ≤ t_1 and $\hat{\alpha}$ is a random variable which is a random parameter of defective rate, uniformly distributed with its *p.d.f.* as ϕ (α) and expected value E(α). ϕ
   $$(\alpha) = \begin{cases} 50, & 0 \le \alpha \le .02 \\ 0, & \text{otherwise} \end{cases}$$
   Then, E(α) = 0.01 and E (α^2) = 0.00013,
7. The demand rate D(t) is assumed to be a ramp type function of time:
   D(t) = $D_0\,[t-(t-\mu)H(t-\mu)]$, (D_0 > 0 , may be crisp or fuzzy)
   where, H $(t-\mu)$ is the well known Heaviside's function defined as follows:
   $$H\,(t-\mu) = \begin{cases} 1; & t \ge \mu, \\ 0; & t < \mu. \end{cases}$$
8. During the permissible credit period μ, the retailer can deposit generated sales revenue in an interest bearing account. At the end of this fixed period, the difference between sales price and unit cost is retained by the system to meet the day-to-day expenses. And the account is settled and interest charges are payable on the unsold items in the stock.

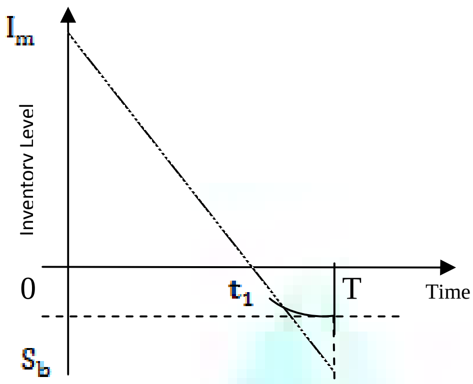
NOTATIONS

T the cycle length
t_1 the length of time in the cycle when on hand inventory level reaches to zero
μ the permissible credit period for settling the account
I(t) the inventory level at time t of the cycle ,
L(t) the amount of lost sale at time t during the time interval [t_1, T]

I_m the maximum inventory level for the cycle
Q^* the optimal order quantity in the cycle i.e. $Q^* = I_{m^*} + S_{b^*}$
S_b the maximum shortage quantity for the cycle
i the inflation rate, may be crisp or fuzzy
r the discount rate , may be crisp or fuzzy
R the net discount rate of inflation i.e. R = r –i , may be crisp or fuzzy
$\ddot{C}_o$ the imprecise order cost per order
$\ddot{C}_h$ the imprecise holding cost per unit per unit time
$\ddot{C}_2$ the imprecise backlogging cost per unit per unit time
$\ddot{C}_p$ and C_p the respective imprecise and crisp purchasing cost per unit per unit time
$\ddot{C}_L$ and C_L the respective imprecise and crisp unit cost of lost sales. Note that if the objective is to minimize the cost, then $C_L > C_p$, (c.f. Chang et. al.,2010)
$\ddot{P}$ and P the respective imprecise and crisp selling price per unit with $(P > C_p)$
i_c the interest charged per monetary unit in stock per unit time by the supplier
i_e the interest earned per monetary unit per unit time by the retailer, where $i_e < i_c$
TC_o the ordering cost
TC_p the purchasing cost
TC_h the holding cost
TC_d the deterioration cost
TC_s the shortage cost
TC_l the lost sale cost
$TC(t_1)$ the total relevant inventory cost for μ=t_1
$TC_1(t_1)$ the total relevant inventory cost for μ<t_1
$TC_2(t_1)$ the total relevant inventory cost for μ>t_1
$E(TC(t_1))$ the expected value of $TC(t_1)$
E ($TC_1(t_1)$) the expected value of $TC_1(t_1)$
E ($TC_2(t_1)$) the expected value of $TC_2(t_1)$

MATHEMATICAL MODEL AND SOLUTION

FIGURE 1 (a): THE GENERAL GRAPHIC REPRESENTATION OF INVENTORY LEVEL WITH PARTIAL BACKLOGGING



The inventory level at time t during the time interval $[0, t_1]$ is given by the differential equation as follows:

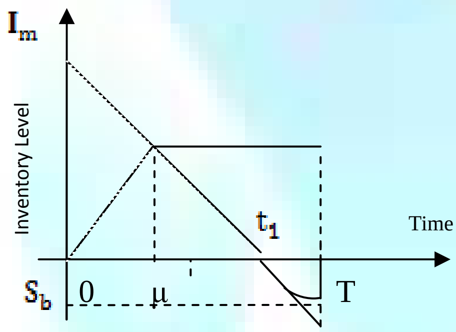
$$dI(t)/dt + \ddot{\theta}(t)\, I(t) = -D(t), \quad 0 \le t \le t_1 \tag{1}$$

The shortage level at time t during the time interval $[t_1, T]$ is given by the following differential equation as

$$dI(t)/dt = D(t)\, B(T-t), \quad t_1 \le t \le T \tag{2}$$

Case -1 ($\mu \le t_1$): The inventory model for payment before depletion with ramp-type demand D(t) is shown in **figure 1(b)**.

FIGURE 1 (b): THE GRAPHIC REPRESENTATION OF INVENTORY LEVEL WHEN $\mu \le t_1$.



The above differential equations (1), (2) in this case are considered as follows:

$$dI(t)/dt + \hat{\alpha}\beta t^{\beta-1} I(t) = -D_0\, t, \quad 0 \le t \le \mu \tag{3}$$

$$dI(t)/dt + \hat{\alpha}\beta t^{\beta-1} I(t) = -D_0\, \mu, \quad \mu \le t \le t_1 \tag{4}$$

$$dI(t)/dt = D_0\, \mu e^{-\delta(T-t)}, \quad t_1 \le t \le T \tag{5}$$

Furthermore, by using the conditions $I(0) = I_m$ and $I(t_1) = 0$, the solutions of equations (3),(4) and (5) are respectively given by

$$I(t) = I_m e^{-\hat{\alpha}t^\beta} - D_0 e^{-\hat{\alpha}t^\beta} \int_0^t x e^{\hat{\alpha}x^\beta} dx, \quad 0 \le t \le \mu \tag{6}$$

$$I(t) = D_0\mu\left(t_1 + \hat{\alpha} t_1^{\beta+1}/(\beta+1)\right)e^{-\hat{\alpha}t^\beta} - D_0\mu e^{-\hat{\alpha}t^\beta}\left(t + \hat{\alpha}t^{\beta+1}/(\beta+1)\right), \quad \mu \le t \le t_1 \tag{7}$$

$$I(t) = (D_0\mu/\delta)\left[e^{-\delta(T-t)} - e^{-\delta(T-t_1)}\right], \quad t_1 \le t \le T \tag{8}$$

By equality conditions at μ , from equations (6) and (7), we get

$$I_m = D_0\left[\int_0^\mu x e^{\hat{\alpha}x^\beta} dx + \mu \int_\mu^{t_1} e^{\hat{\alpha}x^\beta} dx\right] \tag{9}$$

and the amount of lost sale at time t during the time interval $[t_1, T]$ is

$$L(t) = D_0\mu \int_{t_1}^t \left(1 - e^{-\delta(T-x)}\right)dx = D_0\mu\left[t - t_1 - (1/\delta)\left\{e^{-\delta(T-t)} + e^{-\delta(T-t_1)}\right\}\right], t_1 \le t \le T \tag{10}$$

Let S_b be the minimum shortage quantity in the cycle, from equation (8) we get

$$S_b = I(T) = (D_0\mu/\delta)\left[1 - e^{-\delta(T-t_1)}\right] \tag{11}$$

The values of all the costs in the entire time horizon are as follows:

$$TC_o = \ddot{C}_o \tag{12}$$

$TC_s = \tilde{C}_p I_m + \tilde{C}_p S_b$

$$= \tilde{C}_p D_o \left[\int_0^{\mu} x e^{\tilde{a}x^{\beta}} dx + \mu \int_{\mu}^{t_1} e^{\tilde{a}x^{\beta}} dx\right] + \tilde{C}_p \,(D_o\mu/\delta)\left[1 - e^{-\delta(T-t_1)}\right] \quad (13)$$

$$TC_h = \left[\tilde{C}_h D_o \left[\int_0^{\mu} e^{-\tilde{a}t^{\beta}} \left[\int_t^{\mu} x e^{\tilde{a}x^{\beta}} dx + \mu \int_{\mu}^{t_1} e^{\tilde{a}x^{\beta}} dx\right] e^{-Rt} dt\right] + \mu \tilde{C}_h D_o \int_{\mu}^{t_1} \left[e^{-\tilde{a}t^{\beta}} \int_t^{t_1} e^{\tilde{a}x^{\beta}} dx\right] e^{-Rt} dt\right]$$

$$= \left[\tilde{C}_h D_o \left[\int_0^{\mu} e^{-\tilde{a}t^{\beta}} \left[\int_t^{\mu} x e^{\tilde{a}x^{\beta}} dx + \mu \int_{\mu}^{t_1} e^{\tilde{a}x^{\beta}} dx\right] e^{-Rt} dt\right] + \mu \tilde{C}_h D_o \int_{\mu}^{t_1} \left[e^{-\tilde{a}t^{\beta}} \int_t^{t_1} e^{\tilde{a}x^{\beta}} dx\right] e^{-RT} dt\right] \quad (14)$$

$$TC_d = \tilde{C}_p D_o \left[\left\{\int_0^{\mu} x e^{\tilde{a}x^{\beta}} dx + \mu \int_{\mu}^{t_1} e^{\tilde{a}x^{\beta}} dx\right\} + (e^{-R\mu} - 1)/R^2 + \mu e^{-Rt_1}/R\right]$$

$$= \tilde{C}_p D_o \left[\left\{\int_0^{\mu} x e^{\tilde{a}x^{\beta}} dx + \mu \int_{\mu}^{t_1} e^{\tilde{a}x^{\beta}} dx\right\} + (e^{-R\mu} - 1)/R^2 + \mu e^{-Rt_1}/R\right], \quad (15)$$

$$TC_s = \tilde{C}_s \left(\int_{t_1}^{T} I(t) e^{-Rt}\, dt\right) = \frac{\tilde{C}_s D_o \mu}{R}\left[\frac{e^{(R-\delta)(T-t_1)} - 1}{R-\delta} + \frac{e^{-\delta(T-t_1)} - 1}{\delta}\right], \quad (16)$$

$$TC_L = \left[\tilde{C}_L D_o \mu \int_{t_1}^{T} e^{-Rt}\left(1 - e^{-\delta(T-t)}\right) dt\right] = \tilde{C}_L D_o \mu \left[\frac{e^{R(T-t_1)} - 1}{R} + \frac{1 - e^{(R-\delta)(T-t_1)}}{R-\delta}\right], \quad (17)$$

The interest earned by the retailer during the time interval [0, μ] due to the deposition of the sold revenue into an interest earning account at the rate i_e in the entire time horizon is as follows:

$$IE_1 = \tilde{P} D_o i_e \left[\int_0^{\mu} (\mu - t) t e^{-Rt} dt + \mu \int_{\mu}^{t_1} (t_1 - t) e^{-Rt} dt\right] = \tilde{P} D_o i_e \left[\int_0^{\mu} (\mu - t) t e^{-Rt} dt + \mu \int_{\mu}^{t_1} (t_1 - t) e^{-Rt}\right] \quad (18)$$

The interest charged by the supplier from the time μ onwards for the unsold items at the rate i_c is

$$IC_1 = \tilde{C}_p D_o \mu i_c \left[\left(t_1 + \tilde{a}\frac{t_1^{\beta+1}}{\beta+1}\right) \int_{\mu}^{t_1} e^{-\tilde{a}t^{\beta} - Rt} dt - \int_{\mu}^{t_1} e^{-\tilde{a}t^{\beta} - Rt}\left(t + \tilde{a}\frac{t^{\beta+1}}{\beta+1}\right) dt\right]$$

$$= \tilde{C}_p D_o \mu i_c \left[\left(t_1 + \tilde{a} t_1^{\beta+1}/(\beta+1)\right) \int_{\mu}^{t_1} e^{-\tilde{a}t^{\beta} - Rt} dt - \int_{\mu}^{t_1} e^{-\tilde{a}t^{\beta} - Rt}\left(t + \tilde{a} t^{\beta+1}/(\beta+1)\right) dt\right], \quad (19)$$

Hence, the value of the total relevant inventory cost in the entire time horizon is

$$TC_1(t_1) = TC_o + TC_p + TC_h + TC_d + TC_s + TC_L + IC_1 - IE_1 \quad (20)$$

Substituting equations (12) – (19) into equation (20) we obtain

$$TC_1(t_1) = \tilde{C}_o + \tilde{C}_p D_o \left[\int_0^{\mu} x e^{\tilde{a}x^{\beta}} dx + \mu \int_{\mu}^{t_1} e^{\tilde{a}x^{\beta}} dx\right] + \tilde{C}_p \frac{D_o \mu}{\delta}\left[1 - e^{-\delta(T-t_1)}\right] + \left[\tilde{C}_h D_o \left[\int_0^{\mu} e^{-\tilde{a}t^{\beta}} \left[\int_t^{\mu} x e^{\tilde{a}x^{\beta}} dx + \mu \int_{\mu}^{t_1} e^{\tilde{a}x^{\beta}} dx\right] e^{-Rt} dt\right] + \mu \tilde{C}_h D_o \int_{\mu}^{t_1} \left[e^{-\tilde{a}t^{\beta}} \int_t^{t_1} e^{\tilde{a}x^{\beta}} dx\right] e^{-RT} dt\right] +$$
$$\tilde{C}_p D_o \left[\left\{\int_0^{\mu} x e^{\tilde{a}x^{\beta}} dx + \mu \int_{\mu}^{t_1} e^{\tilde{a}x^{\beta}} dx\right\} + \frac{e^{-R\mu} - 1}{R^2} + \frac{\mu e^{-Rt_1}}{R}\right] + D_o \mu \left[\left(\frac{\tilde{C}_s}{R} - \tilde{C}_L\right) \frac{e^{(R-\delta)(T-t_1)} - 1}{R-\delta} + \frac{\tilde{C}_s}{R}\frac{e^{-\delta(T-t_1)} - 1}{\delta} + \tilde{C}_L \frac{e^{R(T-t_1)} - 1}{R}\right]$$
$$+ \tilde{C}_p D_o \mu i_c \left[\left(t_1 + \tilde{a}\frac{t_1^{\beta+1}}{\beta+1}\right) \int_{\mu}^{t_1} e^{-\tilde{a}t^{\beta} - Rt} dt - \int_{\mu}^{t_1} e^{-\tilde{a}t^{\beta} - Rt}\left(t + \tilde{a}\frac{t^{\beta+1}}{\beta+1}\right) dt\right] - \tilde{P} D_o i_e \left[\int_0^{\mu} (\mu - t) t e^{-Rt} dt + \mu \int_{\mu}^{t_1} (t_1 - t) e^{-Rt}\right], \quad (21)$$

Now expected value of $TC_1(t_1)$, neglecting $E(\alpha^2)$ in onwards terms, is :

$$E(TC_1(t_1)) = \tilde{C}_o + \tilde{C}_p D_o \left[\int_0^{\mu} x(1 + E(\alpha)x^{\beta}) dx + \mu \int_{\mu}^{t_1} (1 + E(\alpha)x^{\beta}) dx\right] + \tilde{C}_p \,(D_o\mu/\delta)\left[1 - e^{-\delta(T-t_1)}\right] +$$
$$\left[\tilde{C}_h D_o \left[\int_0^{\mu} (1 - E(\alpha)t^{\beta})\left[\int_t^{\mu} x(1 + E(\alpha)x^{\beta}) dx + \mu \int_{\mu}^{t_1} (1 + E(\alpha)x^{\beta}) dx\right] e^{-Rt} dt\right] + \mu \tilde{C}_h D_o \int_{\mu}^{t_1} \left[(1 - E(\alpha)t^{\beta}) \int_t^{t_1} (1 + E(\alpha)x^{\beta}) dx\right] e^{-RT} dt\right] +$$
$$\tilde{C}_p D_o \left[\left\{\int_0^{\mu} x(1 + E(\alpha)x^{\beta}) dx + \mu \int_{\mu}^{t_1} (1 + E(\alpha)x^{\beta}) dx\right\} + \frac{e^{-R\mu} - 1}{R^2} + \frac{\mu e^{-Rt_1}}{R}\right] + D_o \mu \left[\left(\frac{\tilde{C}_s}{R} - \tilde{C}_L\right) \frac{e^{(R-\delta)(T-t_1)} - 1}{R-\delta} + \frac{\tilde{C}_s}{R}\frac{e^{-\delta(T-t_1)} - 1}{\delta} + \tilde{C}_L \frac{e^{R(T-t_1)} - 1}{R}\right]$$
$$+ \tilde{C}_p D_o \mu i_c \left[\left(t_1 + E(\alpha)\frac{t_1^{\beta+1}}{\beta+1}\right) \int_{\mu}^{t_1} (1 - E(\alpha)t^{\beta}) e^{-Rt} dt - \int_{\mu}^{t_1} (1 - E(\alpha)t) e^{-Rt}\left(t + E(\alpha)\frac{t^{\beta+1}}{\beta+1}\right) dt\right] - \tilde{P} D_o i_e \left[\int_0^{\mu} (\mu - t) t e^{-Rt} dt + \mu \int_{\mu}^{t_1} (t_1 - t) e^{-Rt}\right]$$

(22) Following Grzegorzewski [2008] and Maity et. al. [2005],the fuzzy numbers are transformed to interval numbers and by using Lemma 1 **(Appendix A)**, the expression (22) is minimized as

Minimize $[E(TC_{1L}(t_1)), E(TC_{1R}(t_1))]$ (23)

where $E(TC_{1L}(t_1)) = \{E(TC_1(t_1))$ with cost $C_{oL}, C_{pL}, C_{hL}, C_{sL}, C_{LL}, P_L\}$ (24)

and $E(TC_{1R}(t_1)) = \{E(TC_1(t_1))$ with cost $C_{oR}, C_{pR}, C_{hR}, C_{sR}, C_{LR}, P_R\}$ (25)

In the case of minimization, multi-optimization problem, is formulated in a conservative sense as

Minimize $[E(TC_{1C}(t_1)), E(TC_{1R}(t_1))]$ (26)

where, $E(TC_{1C}(t_1)) = [E(TC_{1L}(t_1)) + E(TC_{1R}(t_1))]/2$, (27)

The interval optimization problem (26) is a multi-objective problem which is converted to a single-objective problem by using the weighted sum method with weights w_1 and w_2 as

Minimize $E(TC_{1CR}(t_1)) = [w_1 E(TC_{1C}(t_1)) + w_2 E(TC_{1R}(t_1))]$, (28)

where, $w_1 + w_2 = 1$, with proper choice of $w_1, w_2 > 0$

THE SINGLE-OBJECTIVE PROBLEM WHICH IS MINIMIZED AS FOLLOWS

There is one variable in the present value of the total inventory cost $E(TC_{1CR}(t_1))$, that is the time t_1, $t_1 \le t \le T$ which is a continuous variable. The condition for $E(TC_{1CR}(t_1))$ to be minimized is that $dE(TC_{1CR}(t_1))/dt_1 = 0 = g_1(t_1)$, (say). Consequently, we obtain

$$g_1(t_1) = C_p D_o \mu (1 + E(\alpha) t_1^{\beta}) + \mu\, C_h D_o (1 + E(\alpha) t_1^{\beta}) \int_0^{t_1} (1 - E(\alpha) t^{\beta}) e^{-Rt} dt +$$

$C_p D_0 \mu\left[(1+E(\alpha)t_1^{\beta}) - e^{-Rt_1}\right] - D_0\mu\left[\left(C_p - (C_s/R)\right)e^{-\delta(T-t_1)} + \left((C_s/R) - C_L\right)e^{(R-\delta)(T-t_1)} + C_L e^{R(T-t_1)}\right] +$

$C_p D_0 i_c \mu\left(1+\alpha t_1^{\beta}\right)\int_{\mu}^{t_1}(1-E(\alpha)t^{\beta})e^{-Rt}dt - PD_0 i_e \mu \int_{\mu}^{t_1} e^{-Rt}dt = 0,$ (29)

It is verified that $d^2E\left(TC_{1CR}(t_1)\right)/dt_1^2 > 0.$ Putting $t_1 = \mu$ in equation (29), we get

$g_1(\mu) = C_p\left(1+E(\alpha)\mu^{\beta}\right) + C_h e^{E(\alpha)\mu^{\beta}}\int_0^{\mu}(1-E(\alpha)t^{\beta})e^{-Rt}dt + C_p\left[(1+E(\alpha)\mu^{\beta}) - e^{-R\mu}\right] - \left[\left(C_p - (C_s/R)\right)e^{-\delta(T-\mu)} + \left((C_s/R) - C_L\right)e^{(R-\delta)(T-\mu)} +\right.$
$\left. C_L e^{R(T-\mu)}\right]e^{-RT}$

(30)

where, $C_o, C_p, C_h, C_s, C_L, P$ are crisp value (c.f. **Appendix A**) .

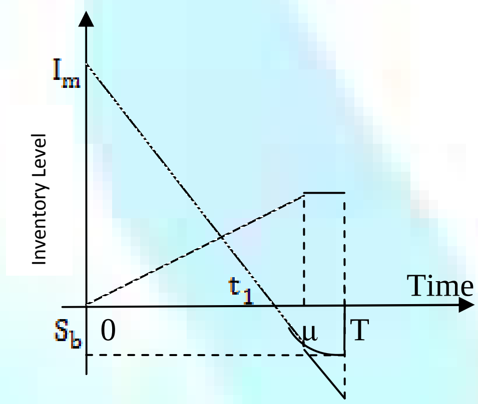
Case – 2 ($\mu > t_1$) : The inventory model for payment after depletion with ramp-type demand D(t) is shown in **figure 1(c).** The differential equations (1), (2) in this case are considered as follows:

$dI(t)/dt + \tilde{\alpha}\beta\, t^{\beta-1}\, I(t) = -D_0\, t, \quad 0 \le t \le t_1$ (31)

$dI(t)/dt = D_0 t e^{-\delta(\mu-t)}, \quad t_1 \le t \le \mu$ (32)

$dI(t)/dt = D_0 \mu e^{-\delta(T-t)}, \quad \mu \le t \le T$ (33)

FIGURE 1 (c): THE GRAPHIC REPRESENTATION OF INVENTORY LEVEL WHEN $\mu > t_1$.



Furthermore, by using the conditions $I(0) = I_m$ and $I(t_1) = 0$, the solutions of equations (31),(32) and (33) are respectively given by

$I(t) = D_0 e^{-\tilde{a}t^{\beta}}\int_t^{t_1} x e^{\tilde{a}x^{\beta}}dx, \quad 0 \le t \le t_1$ (34)

$I(t) = D_0 \int_{t_1}^{t} t e^{-\delta(\mu-t)}dt, \quad t_1 \le t \le \mu \quad = (D_0\mu/\delta)$
$\left[te^{-\delta(\mu-t)} - t_1 e^{-\delta(\mu-t_1)}\right] - (D_0/\delta^2)\left[e^{-\delta(\mu-t)} - e^{-\delta(\mu-t_1)}\right],$ (35)

$I(t) = (D_0/\delta)\left[e^{-\delta(T-t)} - e^{-\delta(T-\mu)}\right] + I(\mu), \quad \mu \le t \le T$ (36)

$= (D_0/\delta)\left(\mu - t_1 e^{-\delta(\mu-t_1)}\right) + (D_0\mu/\delta)\left(e^{-\delta(T-t)} - e^{-\delta(T-\mu)}\right) - (D_0/\delta^2)\left[1 - e^{-\delta(\mu-t_1)}\right],$ (37)

From equation (34), we get, $I_m = I(0) = D_0\int_0^{t_1} x e^{\tilde{a}x^{\beta}}dx$ (38)

Now, the amount of lost sale at time t during the time interval $[t_1, \mu]$ and $[\mu, T]$ are respectively

$L_1(t) = D_0\int_{t_1}^{t} x\left(1 - e^{-\delta(\mu-x)}\right)dx, \quad t_1 \le t \le \mu$
$= D_0\left[(t^2 - t_1^2)/2 - (1/\delta)\left\{te^{-\delta(\mu-t)} + t_1 e^{-\delta(\mu-t_1)}\right\} + (1/\delta^2)\left\{e^{-\delta(\mu-t)} - e^{-\delta(\mu-t_1)}\right\}\right],$ (39)

and, $L_2(t) = D_0\mu\int_{\mu}^{t}\left(1 - e^{-\delta(T-x)}\right)dx = D_0\mu\left[(t-\mu) - (1/\delta)\left\{e^{-\delta(T-t)} - e^{-\delta(T-\mu)}\right\}\right], \mu \le t \le T$ (40)

From equation (37) we get
$S_b = I(T) = (D_0/\delta)\left(\mu\left(1 - e^{-\delta(T-\mu)}\right) + \left(\mu - t_1 e^{-\delta(\mu-t_1)}\right) - (1/\delta)\left[1 - e^{-\delta(\mu-t_1)}\right]\right],$ (41)

The values of all the costs in the entire time horizon are

TC_0= same as in equation(12). (42)

$TC_p = \tilde{C}_p I_m + \tilde{C}_p S_b$
$= \tilde{C}_p D_0\left[\int_0^{t_1} x e^{\tilde{a}x^{\beta}}dx\right] + \tilde{C}_p D_0\left[\frac{\mu}{\delta}\left(1 - e^{-\delta(T-\mu)}\right) + \frac{1}{\delta}\left(\mu - t_1 e^{-\delta(\mu-t_1)}\right) - \frac{1}{\delta^2}\left[1 - e^{-\delta(\mu-t_1)}\right]\right],$

(43)

$TC_h = \tilde{C}_h D_0\left[\int_0^{t_1} e^{-\tilde{a}t^{\beta}}\left[\int_t^{t_1} x e^{\tilde{a}x^{\beta}}dx\right]e^{-Rt}dt\right]$

$$= \tilde{C}_h D_0 \left[\int_0^{t_1} e^{-\bar{a}t^{\beta}} \left[\int_t^{t_1} x e^{\bar{a}x^{\beta}} dx\right] e^{-Rt} dt\right], \tag{44}$$

$$TC_d = \tilde{C}_p D_0 \left[\int_0^{t_1} x e^{\bar{a}x^{\beta}} dx + (e^{-Rt_1} - 1)/R + (t_1 e^{-Rt_1})/R\right]$$

$$= \tilde{C}_p D_0 \left[\int_0^{t_1} x e^{\bar{a}x^{\beta}} dx + (e^{-Rt_1} - 1)/R + (t_1 e^{-Rt_1})/R\right], \tag{45}$$

$$TC_s = \tilde{C}_s \left[\int_{t_1}^{\mu} I(t) e^{-Rt} dt + \int_{\mu}^{T} I(t) e^{-Rt} dt\right] =$$

=

$$\tilde{C}_s D_0 \left[\mu \int_{\mu}^{T} e^{-\delta(T-t)-Rt} dt - \{\mu(e^{-\delta(T-\mu)}) - (\mu - t_1 e^{-\delta(\mu-t_1)}) + (1/\delta)(1 - e^{-\delta(\mu-t_1)})\} \int_{\mu}^{T} e^{-Rt} dt - \{(t_1 e^{-\delta(\mu-t_1)}) - (1/\delta)(e^{-\delta(\mu-t_1)})\} \int_{t_1}^{\mu} e^{-Rt} dt + \int_{t_1}^{\mu} e^{-\delta(\mu-t)-Rt} dt - (1/\delta) \int_{t_1}^{\mu} e^{-\delta(\mu-t)-Rt} dt\right](1/\delta)$$

, (46)

$$TC_L = \left[\tilde{C}_L D_0 \int_{t_1}^{\mu} t e^{-Rt}\left(1 - e^{-\delta(\mu-t)}\right) dt + \tilde{C}_L D_0 \mu \int_{\mu}^{T} e^{-Rt}\left(1 - e^{-\delta(T-t)}\right) dt\right]$$

$$= \left[\tilde{C}_L D_0 \int_{t_1}^{\mu} t e^{-Rt}\left(1 - e^{-\delta(\mu-t)}\right) dt + \tilde{C}_L D_0 \mu \int_{\mu}^{T} e^{-Rt}\left(1 - e^{-\delta(T-t)}\right) dt\right], \tag{47}$$

$$IE_2 = \tilde{P} D_0 i_e \left[\int_0^{t_1} (t_1 - t) t e^{-Rt} dt + (\mu - t_1) \int_0^{t_1} t\, dt \times \int_{t_1}^{\mu} e^{-Rt} dt\right]$$

$$= \tilde{P} D_0 i_e \left[\int_0^{t_1} (t_1 - t) t e^{-Rt} dt + (\mu - t_1) \int_0^{t_1} t\, dt \times \int_{t_1}^{\mu} e^{-Rt} dt\right], \tag{48}$$

Here, the retailer sells all the monetary units by the end of the cycle time t_1 and pays the supplier in full by the end of the credit period μ. So, $IC_2 = 0$,

(49)

Hence, the value of the total relevant inventory cost in the entire time horizon is

$$TC_2(t_1) = \left(TC_o + TC_p + TC_h + TC_d + TC_s + TC_L + IC_2 - IE_2\right) \tag{50}$$

Substituting equations (12) and (43) – (49) into equation (50) we obtain

$$TC_2(t_1) = \tilde{C}_o + \tilde{C}_p D_0 \left[\int_0^{t_1} x e^{\bar{a}x^{\beta}} dx\right] + (\tilde{C}_p D_0/\delta)\left[\mu\left(1 - e^{-\delta(T-\mu)}\right) + \left(\mu - t_1 e^{-\delta(\mu-t_1)}\right) - (1/\delta)\left[1 - e^{-\delta(\mu-t_1)}\right]\right] + \tilde{C}_h D_0 \left[\int_0^{t_1} e^{-\bar{a}t^{\beta}} \left[\int_t^{t_1} x e^{\bar{a}x^{\beta}} dx\right] e^{-Rt} dt\right] + \left[\int_0^{t_1} x e^{\bar{a}x^{\beta}} dx + (e^{-Rt_1} - 1)/R^2 + t_1 e^{-Rt_1}/R\right] \tilde{C}_p D_0$$

$$+ (\tilde{C}_s D_0/\delta)\left[\mu \int_{\mu}^{T} e^{-\delta(T-t)-Rt} dt - \{\mu e^{-\delta(T-\mu)} - (\mu - t_1 e^{-\delta(\mu-t_1)}) + (1/\delta)(1 - e^{-\delta(\mu-t_1)})\} \int_{\mu}^{T} e^{-Rt} dt - \{(t_1 e^{-\delta(\mu-t_1)}) - (1/\delta)(e^{-\delta(\mu-t_1)})\} \int_{t_1}^{\mu} e^{-Rt} dt + \int_{t_1}^{\mu} e^{-\delta(\mu-t)-Rt} dt - (1/\delta) \int_{t_1}^{\mu} e^{-\delta(\mu-t)-Rt} dt\right] + \left[\tilde{C}_L D_0 \int_{t_1}^{\mu} t e^{-Rt}\left(1 - e^{-\delta(\mu-t)}\right) dt + \tilde{C}_L D_0 \mu \int_{\mu}^{T} e^{-Rt}\left(1 - e^{-\delta(T-t)}\right) dt\right] - \tilde{P} D_0 i_e \left[\int_0^{t_1} (t_1 - t) t e^{-Rt} dt + (\mu - t_1) \int_0^{t_1} t\, dt \times \int_{t_1}^{\mu} e^{-Rt} dt\right], \tag{51}$$

Now, expected value of $TC_2(t_1)$ is

$$E\left(TC_2(t_1)\right) = \tilde{C}_o + \tilde{C}_p D_0 \left[\int_0^{t_1} x(1 + E(\alpha)x^{\beta}) dx\right] + (\tilde{C}_p D_0/\delta)\left[\mu\left(1 - e^{-\delta(T-\mu)}\right) + \left(\mu - t_1 e^{-\delta(\mu-t_1)}\right) - (1/\delta)\left[1 - e^{-\delta(\mu-t_1)}\right]\right] + \tilde{C}_h D_0 \left[\int_0^{t_1} (1 - E(\alpha)t^{\beta}) \left[\int_t^{t_1} x(1 + E(\alpha)x^{\beta}) dx\right] e^{-Rt} dt\right] + \tilde{C}_p D_0 \left[\int_0^{t_1} x(1 + E(\alpha)x^{\beta}) dx + (e^{-Rt_1} - 1)/R^2 + t_1 e^{-Rt_1}/R\right] +$$

$$\tilde{C}_s D_0 \left[\mu \int_{\mu}^{T} e^{-\delta(T-t)-Rt} dt - \{\mu(e^{-\delta(T-\mu)}) - (\mu - t_1 e^{-\delta(\mu-t_1)}) + (1/\delta)(1 - e^{-\delta(\mu-t_1)})\} \int_{\mu}^{T} e^{-Rt} dt - \{(t_1 e^{-\delta(\mu-t_1)}) - (1/\delta)(e^{-\delta(\mu-t_1)})\} \int_{t_1}^{\mu} e^{-Rt} dt + \int_{t_1}^{\mu} e^{-\delta(\mu-t)-Rt} dt - (1/\delta) \int_{t_1}^{\mu} e^{-\delta(\mu-t)-Rt} dt\right](1/\delta) + \left[\tilde{C}_L D_0 \int_{t_1}^{\mu} t e^{-Rt}\left(1 - e^{-\delta(\mu-t)}\right) dt + \tilde{C}_L D_0 \mu \int_{\mu}^{T} e^{-Rt}\left(1 - e^{-\delta(T-t)}\right) dt\right] -$$

$\tilde{P} D_0 i_e \left[\int_0^{t_1} (t_1 - t) t e^{-Rt} dt + (\mu - t_1) \int_0^{t_1} t\, dt \times \int_{t_1}^{\mu} e^{-Rt} dt\right]$, neglecting expected values of $\alpha^2, \alpha^3, \alpha^4, \ldots$,

(52)

Now, the single objective problem E ($TC_{2CR}(t_1)$) obtained from (52) by the procedure similar to case 1 is minimized. Thus,

Minimize E ($TC_{2CR}(t_1)$) = [w_1E ($TC_{2C}(t_1)$) + w_2E ($TC_{2R}(t_1)$)], (53)

and $w_1 + w_2 = 1,\ w_1, w_2 > 0$

The condition for E($TC_{2CR}(t_1)$) to be minimum is that, $dE(TC_{2CR}(t_1))/dt_1$ = 0 = $g_2(t_1)$ (say).

where, $g_2(t_1)$ =

$$C_p e^{E(\alpha)t_1^{\beta}} + C_h e^{E(\alpha)t_1^{\beta}} \int_0^{t_1} e^{-E(\alpha)t^{\beta}-Rt} dt + C_p\left[e^{E(\alpha)t_1^{\beta}} - e^{-Rt_1}\right] - \left[\left(C_p - (C_s/R)\right)e^{-\delta(\mu-t_1)} + \left((C_s/R) - C_L\right)e^{-\delta(\mu-t_1)+R(T-t_1)} + C_L e^{R(T-t_1)}\right]e^{-RT} - Pi_e\left[(\mu - (3/2)t_1)\int_{t_1}^{\mu} e^{-Rt} dt - (\mu - t_1)(t_1/2)e^{-Rt_1}\right]$$

= 0, (54)

It is verified that $d^2E(TC_{2CR}(t_1))/dt_1^2 > 0$.

$$\text{and, } g_2(\mu) = C_p e^{E(\alpha)\mu^{\beta}} + C_h e^{E(\alpha)\mu^{\beta}} \int_0^{\mu} e^{-E(\alpha)t^{\beta}-Rt} dt + C_p\left[e^{E(\alpha)\mu^{\beta}} - e^{-R\mu}\right] - \left[\left(C_p - (C_s/R)\right) + (C_s/R)e^{(R)(T-\mu)}\right]e^{-RT}, \tag{55}$$

For the inventory model, the optimal replenishment time is always attained as t_1^*, where t_1^* is the unique solution for both $g_1(t_1) = 0$ and $g_2(t_1) = 0$. Now, the conditions for $\mu \le t_1^*$ and $t_1^* < \mu$ are verified by using the following two theorems:

Theorem 1: If $g_1(\mu) \le 0$, then there exists a unique solution $t_1^* \ge \mu$, which is the minimum point, where $\mu \le t_1^* < T$, such that $E(TC_{1CR}(t_1^*))$ is obtaining the minimum value. With this value of t_1^*,

$$I_{m^*} = D_0 \left[\int_0^{\mu} x(1+E(\alpha)x^{\beta})dx + \mu\int_{\mu}^{t_1^*}(1+E(\alpha)x^{\beta})dx\right], \tag{56}$$

and, $$Q^* = D_0 \left[\int_0^{\mu} x(1+E(\alpha)x^{\beta})dx + \mu\int_{\mu}^{t_1^*}(1+E(\alpha)x^{\beta})dx\right] + (D_0\mu/\delta)\left[1-e^{-\delta(T)-t_1^*)}\right] \tag{57}$$

Theorem 2: If $g_2(\mu) > 0$, then there exists a unique solution $t_1^* < \mu$, which is the minimum point, where $t_1^* < \mu < T$, such that $E(TC_{2CR}(t_1^*))$ is obtaining the minimum value.

Similarly, as in case-1,we can take (t_1^*) as the optimal solution to $E(TC_{2CR}(t_1))$, and

$$I_{m^*} = D_0 \int_0^{t_1^*} x e^{E(\alpha)x^{\beta}} dx, \tag{58}$$

$$Q^* = D_0 \left[\int_0^{t_1^*} x e^{E(\alpha)x^{\beta}} dx\right] + \frac{D_0\mu}{\delta}\left(1-e^{-\delta(T-\mu)}\right) + \frac{D_0}{\delta}\left(\mu - t_1^* e^{-\delta(\mu - t_1^*)}\right) - \frac{D_0}{\delta^2} \tag{59}$$

MODEL CLASSIFICATION

Here, we have considered two models.

MODEL – 1 (FUZZY STOCHASTIC MODEL WITH CRISP DEMAND AND INFLATION)

Here we Minimize $E(TC_{1CR}(t_1))$ for $\mu < t_1$, Minimize $E(TC_{CR}(t_1))$ for $\mu = t_1$ and Minimize $E(TC_{2CR}(t_1))$ for $\mu > t_1$, with the help of the methods mentioned above.

MODEL –2 (FUZZY STOCHASTIC MODEL WITH FUZZY DEMAND AND INFLATION)

Here, we suppose that $\tilde{D}_0 = (D_{01}, D_{02}, D_{03}, D_{04})$, $\tilde{r} = (r_1, r_2, r_3, r_4)$, $\tilde{i} = (i_1, i_2, i_3, i_4)$, $\tilde{R} = (R_1, R_2, R_3, R_4)$ are non-negative trapezoidal fuzzy numbers. So, fuzzy total relevant inventory costs for $\mu < t_1$, $\mu = t_1$ and $\mu > t_1$ are respectively found by the process used in model – 1 along with function principle as:

$E(\overline{TC_{1CR}(t_1)}) = [E(TC_{1CR}(t_1))_1, E(TC_{1CR}(t_1))_2, E(TC_{1CR}(t_1))_3, E(TC_{1CR}(t_1))_4]$,

$E(TC_{CR}(t_1)) = [E(TC_{CR}(t_1))_1, E(TC_{CR}(t_1))_2, E(TC_{CR}(t_1))_3, E(TC_{CR}(t_1))_4]$,

$E(\overline{TC_{2CR}(t_1)}) = [E(TC_{2CR}(t_1))_1, E(TC_{2CR}(t_1))_2, E(TC_{2CR}(t_1))_3, E(TC_{2CR}(t_1))_4]$.

Now, using Graded Mean Integration Representation method (c.f. Chen et.al.(2005)), the possibilistic mean value of the fuzzy total relevant inventory costs are expressed by $P(E(TC_{1CR}(t_1)))$, $P(E(TC_{CR}(t_1)))$ and $P(E(TC_{2CR}(t_1)))$.

IMPERFECT QUALITY WITH LEARNING EFFECTS

Jaber et al. [17], have considered learning effects on an economic order quantity for items with imperfect quality. However $\tilde{\alpha}$ is replaced with α (k) which is the percentage of defective per shipment k in $\tilde{\theta}$ (t). For example α (k) is expressed using the S-shaped logistic learning curve model as follows:
α (k) = $\frac{\alpha_1}{\alpha_2 + e^{\alpha_3 k}}$ where $\alpha_1, \alpha_2, \alpha_3$ are positive model parameters, k is the cumulative number of shipments and α (k) is the percentage defective per shipment k.

NUMERICAL EXAMPLES (USING LINGO SOFTWARE)

Now, we try to verify our models using numerical examples for two cases:

(A) Without Learning Effects, **(B)** With Learning Effects.

(A) WITHOUT LEARNING EFFECTS

OPTIMUM RESULTS OF MODEL- 1

To illustrate the model -1, let us consider the following parametric values:

R= 0.15, T = 1 year, β = 3, δ = 0.02, D_0 = 500, i_c= 0.08$, i_e= 0.13$, α_1= 70.067, α_2= 7005.50, α_3 = 0.7932, $\ddot{C}_o$= (195,245,275), $\ddot{C}_s$= (1,2,5), $\check{C}_p$= (1,3,9), $\ddot{C}_h$= (0.5,1.50,2.50), $\dot{C}_L$= (12,16,28), $\tilde{P}$= (6,12,22). Weighted interval numbers with weight q = 0.5 are $(C_{oL}, C_{oR}) = (220,260)$, $(C_{sL}, C_{sR}) = (1.5, 3.5)$, (C_{pL}, C_{pR}) = (2,6), (C_{hL}, C_{hR}) = (1,2), (C_{LL}, C_{LR}) =(14,22), (P_L, P_R) = (9,17). C_o= 250\$/order, C_p= 5\$/unit, C_h=1.75\$/unit/year, C_s= 3\$/unit/year, C_L= 20 \$/unit/year, P = \$15 **(c.f. Appendix A)**, and $w_1 = w_2 = 0.5$.

Example 1; when ($\mu = t_1 - 0.12 < t_1$): The above parametric data satisfy theorem - 1. The optimal results are $t_1^* = 0.430009$ years, $Q^* = 130.4894$ and $E(TC_1(t_1^*)) = \$\ 1567.173$.

Example 2; when ($\mu = t_1 + 0.12 > t_1$): The above parametric data satisfy Theorem -2. The optimal results are: t_1^* = 0.388964 years, $Q^* = 189.0874$ and $E(TC_2(t_1^*)) = \$\ 79548.60$.

Example 3: when ($\mu = t_1$). The above parametric data satisfy Theorem -1. The optimal results are $t_1^* = 0.418578$ years, $Q^* = 164.7955$ and $E(TC(t_1^*)) = \$\ 1682.576$.

SENSITIVITY ANALYSIS OF MODEL-1

In this section, we perform sensitivity analysis by changing system parameters α, β, δ, R, D_0, one at a time by -20% , -50% , 20% and 50% to investigate their influence on minimum total relevant cost, the optimal order quantity and replenishment number for $\mu < t_1$, $\mu > t_1$ and $\mu = t_1$ shown respectively in examples -1, 2 and 3 of TABLE-1 . The estimated values of the minimum total relevant costs for the three examples can be represented by $E(TC_1(t_1^*))'$, $E(TC_2(t_1^*))'$, $E(TC(t_1^*))'$ respectively, and the optimal order quantity can be represented by Q' Let, for the each minimum total relevant cost, estimated value / optimal value = z and Q' / Q^* = y.

TABLE 1: SENSITIVITY ANALYSIS OF MODEL-1

Parameter		Percentage of under and over estimated parameters of Example -1,Example- 2 and example -3								
		Example -1			Example- 2			example -3		
		-50 , -20	0	+20 , +50	-50 , -20	0	+20 , +50	-50 , -20	0	+20 , +50
E(α)	y	1.0020,1.0008	1	0.9992,0.9968	1.0007,1.0003	1	0.9997,0.9993	1.0012, 1.0005	1	0.9995, 0.9987
	z	1.0016,1.0007	1	0.9993,0.9974	0.9999,0.9999	1	1.0000,1.0001	1.0015, 1.0006	1	0.9994, 0.9985
β	y	0.9903,0.9974	1	1.0016,1.0028	0.9956,0.9989	1	1.0006,1.0011	0.9937, 0.9984	1	1.0010, 1.0017
	z	0.9923,0.9979	1	1.0013,1.0023	1.0006,1.0001	1	0.9999,0.9998	0.9928, 0.9981	1	1.0012, 1.0021
δ	y	0.9655,0.9865	1	1.0131,1.0312	0.9979,0.0002	1	1.0008,1.0020	0.9773, 0.9911	1	1.0086, 1.0210
	z	0.9664,0.9867	1	1.0131,1.0323	1.9038,1.2247	1	0.8515,0.7049	0.9652, 0.9862	1	1.0136, 1.0335
R	y	1.2621,1.1057	1	0.8943,0.7377	1.1248,1.0514	1	0.9474,0.8678	1.1716, 1.0699	1	0.92950.8239
	z	1.2450,1.0920	1	0.9151,0.7996	2.1585,1.2557	1	0.8377,0.6781	1.2407, 1.0902	1	0.9170, 0.8044
D_0	y	0.4999,0.7999	1	1.1999,1.4999	0.5000,0.8000	1	1.2000,1.5000	0.5000, 0.7999	1	1.2000,1.5000
	z	0.5798,0.8319	1	1.1681,1.4202	0.4971,0.7988	1	1.2012,1.5029	0.5744, 0.8297	1	1.1703, 1.4256

For $\mu < t_1$: The optimal order quantity and minimum total relevant cost increase as β, D_0, δ increase. But they decrease as R , E(α) increase. They are more sensitive on the change in E(α), R , β, D_0, δ to other parameters.

For $\mu > t_1$: The optimal order quantity increases as β , D_0, δ increases. But it decreases as E(α), R increase. It is more sensitive on the change in R, D_0 to other parameters. The minimum total relevant cost increases as E(α), D_0 increases. But it decreases as β , R, δ increases. It is more sensitive on the change in δ, R, D_0 to the other parameters.

For, ($\mu = t_1$): The optimal order quantity and minimum total relevant cost increase as β , δ , D_0 increases. But they decrease as E(α), R increase. They are more sensitive on the change in D_0 to other parameters.

OPTIMUM RESULTS OF MODEL- 2

$\tilde{D}_0 = (400,450,550,600)$ and $\tilde{R} = (0.13,0.14,0.16,0.17)$. And other parameters are as in model-1.

Example – 1;($\mu = t_1 - 0.12$): t_1^* = (0.45718,0.443469,0.416804,0.403850), Q^* = (111.7530,121.5831,138.4752,145.5466), $E(TC_1(t_1^*))^*$ = (1379.64,1477.573,1648.779,1722.710). $P(t_1^*)$ =0.430263, $P(Q^*)$ = 129.5694, $P(E(TC_1(t_1^*))^*)$ =1559.176.

Example – 2; ($\mu = t_1 +$ 0.12): t_1^* = (0.416088,0.402396,0.375789,0.362870), Q^* = (156.4809,173.1227,204.3716,218.9737), $E(TC_2(t_1^*))^*$ = (73426.54,76653.16,82172.72,84568.36). $P(t_1^*)$ = 0.38922, $P(Q^*)$ =188.4072, $P(E(TC_2(t_1^*))^*)$ =79274.44.

Example – 3;($\mu = t_1$): t_1^* = (0.445578,0.431950,0.405458,0.392590), Q^* = (137.9916,151.7858,177.0202,188.4618), $E(TC(t_1^*))^*$ = (1475.947,1583.627,1773.165,1855.743). $P(t_1^*)$ =0.418831, $P(Q^*)$ =164.0109, $P(E(TC(t_1^*))^*)$ = 1674.212.

On comparing the objective results of the crisp inflation and demand with fuzzy inflation and demand (without learning effect); it is clear that the objective results with fuzzy inflation and demand are better than the crisp one in all the three examples.

(B) WITH LEARNING EFFECTS

OPTIMUM RESULTS OF MODEL- 1

TABLE 2: DIFFERENT NUMBER OF SHIPMENTS WITH LEARNING EFFECT FOR EXAMPLE-1, 2 AND 3

	Example - 1; $(\mu = t_1 - 0.12)$			Example – 2; $(\mu = t_1 + 0.12)$			Example -3; $(\mu = t_1)$		
k	t_1^*	Q^*	$E(TC_1(t_1^*))$	t_1^*	Q^*	$E(TC_2(t_1^*))$	t_1^*	Q^*	$E(TC(t_1^*))$
1	0.430010	130.4894	1567.174	0.388964	189.0875	79548.60	0.418577	164.7956	1682.577
2	0.430010	130.4896	1567.176	0.388965	189.0876	79548.59	0.418578	164.7957	1682.579
3	0.430012	130.4901	1567.180	0.388966	189.0878	79548.58	0.418579	164.7960	1682.583
4	0.430014	130.4910	1567.190	0.388968	189.0883	79548.55	0.418582	164.7968	1682.592
5	0.430021	130.4932	1567.211	0.388972	189.0894	79548.49	0.418588	164.7984	1682.612
6	0.430034	130.4978	1567.256	0.388983	189.0919	79548.36	0.418600	164.8020	1682.655
7	0.430064	130.5077	1567.355	0.389005	189.0972	79548.07	0.418627	164.8096	1682.748
8	0.430125	130.5283	1567.559	0.389050	189.1081	79547.47	0.418684	164.8254	1682.942
9	0.430244	130.5683	1567.957	0.389140	189.1294	79546.31	0.418793	164.8562	1683.319
10	0.430447	130.6370	1568.639	0.389292	189.1659	79544.32	0.418980	164.9089	1683.964
11	0.430731	130.7326	1569.589	0.389505	189.2167	79541.55	0.419241	164.9824	1684.863
12	0.431030	130.8333	1370.591	0.389729	189.2701	79538.62	0.419515	165.0598	1685.811
13	0.431265	130.9126	1571.380	0.389905	189.3122	79536.31	0.419732	165.1207	1686.556
14	0.431411	130.9619	1571.870	0.390014	189.3383	79534.87	0.419866	165.1586	1687.020
15	0.431489	130.9881	1572.131	0.390073	189.3522	79534.10	0.419938	165.1788	1687.267
16	0.431527	131.0010	1572.259	0.390101	189.3590	79533.73	0.419973	165.1887	1687.388
17	0.431545	131.0070	1572.319	0.390115	189.3622	79533.55	0.419989	165.1933	1687.445
18	0.431554	131.0098	1572.347	0.390121	189.3637	79533.47	0.419997	165.1954	1687.471
19	0.431557	131.0111	1572.360	0.390124	189.3643	79533.43	0.420000	165.1964	1687.483
20	0.431559	131. 0116	1572.365	0.390125	189.3646	79533.41	0.420002	165.1969	1687.488
21	0.431560	131. 0119	1572.368	0.390125	189.3648	79533.41	0.420003	165.1972	1687.491
22	0.431560	131. 0120	1572.369	0.390125	189.3648	79533.40	0.42003	165.1972	1687.492
23	0.431560	131. 0121	1572.370	-	-	-	-	-	-
24	0.431560	131. 0121	1572.370	-	-	-	-	-	-

It is clear from TABLE 2 that optimum quantity is increased with learning effect in all the three examples. In example 1 it increased from 130.4894 to 131.0121 after the 24 consecutive shipments. After that it is being constant. In example 2 it increased from 189.0875 to 189.3648 after the 22 consecutive shipments. After that it is being constant. In example 3 it increased from 164.7956 to 165.1972 after the 22 consecutive shipments. After that it is being constant.So, on applying the learning effect in both the models on deterioration rate to reduce the percentage of defective, it is analyzed that percentage of optimum quantity is increasing in each shipment due to this reduction.

Learning curves of all the three examples of model – 1 are shown in Figure-3 for optimum order quantity against number of shipments.

Figure -3(a): *Example – 1;* $(\mu = t_1 - 0.7425)$

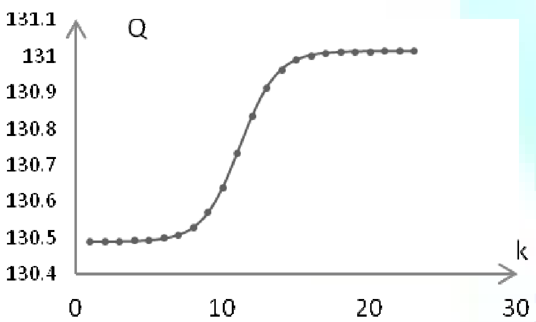


Figure -3(b): *Example – 2;* $(\mu = t_1 + 0.12)$

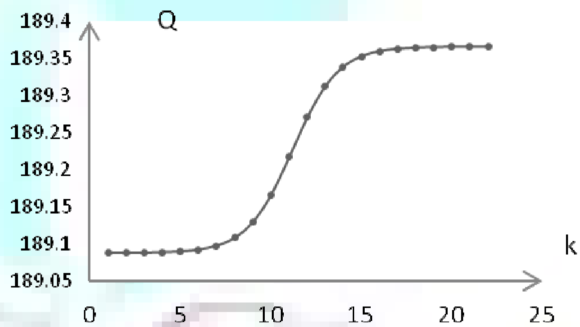
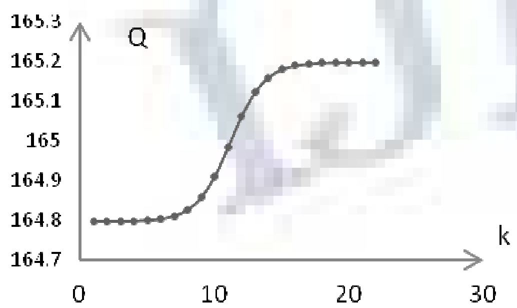


Figure -3(c): *Example – 3;* $(\mu = t_1)$



Optimum quantity of model-1 follows S- shaped learning curve when learning effect applied in each consecutive planning horizon. It is being cleared from Figure 3 (a), (b), (c) which the percentage of order quantity is increasing by reducing the deterioration rate with learning effect in each shipment in all the three examples.

OPTIMUM RESULTS OF MODEL- 2

$\tilde{D}_0 = (400,450,550,600)$ and $\tilde{R} = (0.13,0.14,0.16,0.17)$;

Example – 1; ($\mu = t_1 -$ 0.7425): $t_1^* =$ (1.1303,1.0918,0.8482,0.8203), $Q^* =$ (345.84,362.46,112.48,90.88), $E(TC_1(t_1^*))^* =$ (8565.98,8825.29,8040.01,8086.90). $P(t_1^*) =$ 0.9718, $P(Q^*)$ =232.60, $P(E(TC_1(t_1^*))^*)$ = 8397.25.

Example – 2; ($\mu = t_1$+0.12): $t_1^* =$ (0.9733,0.9354 ,0.8622,0.6813), $Q^* =$ (845.63, 926.81, 1072.81, 762.04), $E(TC_2(t_1^*))^* =$ (323223.5,334679.2,350544.8,354198.8). $P(t_1^*)$ = 0.8750, $P(Q^*)$ = 934.49, $P(E(TC_2(t_1^*))^*)$ = 341308.40.

Example – 3; ($\mu = t_1$): $t_1^* =$ (0.3472, 0.3134, 0.2945, 0.2673), $Q^* =$ (82.48, 78.42,91.61,85.21), $E(TC(t_1^*))^* =$ (8653.25, 8909.33, 9221.58, 9294.56). $P(t_1^*) =$ 0.3051, $P(Q^*)$ = 84.62, $P(E(TC(t_1^*))^*)$ = 9034.94.

CONCLUSION

In this article, we develop an inventory model for random Weibull deterioration with ramp – type demand and delay in payments under partial backlogging to determine the optimal order quantity, the minimum value of total relevant cost. All costs are taken as triangular fuzzy numbers. The effects of inflation and time value of money are also considered. Inflation and demand are also taken as crisp/trapezoidal fuzzy numbers. We present Theorem 1 and 2 to find unique solution of the total relevant cost. From the sensitivity analysis it is found that the optimal order quantity is more sensitive on the change in the parameters E(α), R, β, D_0, δ when μ < t_1;on the change in the parameters R, D_0 when μ > t_1; on the change in the parameter D_0 when μ = t_1. The minimum value of total relevant cost is more sensitive on change in the parameters E(α), R , β, D_0 , δ when (μ < t_1); δ, R, D_0 when (μ > t_1); D_0 when (μ = t_1). It helps retailer to make decisions in different replenishment policies. Optimum order quantities of model-1 and 2 in all the three examples are increased by reducing the deterioration rate with learning effects in consecutive planning horizon. Finally, the proposed model can be extended in several ways. For example, we could extend the fuzzy stochastic model to the case of multi cycle model in a planning horizone, fuzzy random planning horizon, multi-items.

APPENDIX A

1. INTERVAL ARITHMETIC:

Lemma 1. If f is a continuous interval-valued function of a real variable x in [a , b] , then there is a pair of continuous real-valued functions f_1 , f_2 such that f(x) =[f_1(x) , f_2(x)] and the integral of f is equivalent to, $\int_{[a,b]} f(x')dx' = \left[\int_{[a,b]} f_1(x')dx' + \int_{[a,b]} f_2(x'dx')\right]$.

Proof. Maity and Maity [2005] proved this Lemma 1.

2. THE NEAREST INTERVAL APPROXIMATION OF A FUZZY NUMBER:

If $\tilde{A}$ is a fuzzy number with η-cut $[A_L(\eta), A_R(\eta)]$ then according to Grzegorzewsky [2008], the nearest interval approximation of $\tilde{A}$ is $\left[\int_0^1 A_L(\eta)d\eta, \int_0^1 A_R(\eta)d\eta\right]$. Therefore, middle point of the expected interval number considering $\tilde{A}$ = (a_1, a_2, a_3) as a triangular fuzzy number is $[((a_1 + a_2)/2), ((a_2 + a_3)/2)]$ and sometimes its generalization, called weighted expected value, might be interesting. It is given by $[((1-q)a_1 + qa_2), ((1-q)a_2 + qa_3)]$.

3. $\tilde{A} = [A_L, A_R]$, A = $w_1(A_L + A_R)/2 + w_2 A_R$, $w_1 = w_2 = 0.5$, where, $A \to \{C_o, C_p, C_h, C_s, C_L, P\}$.

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