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- Schemenner, R.W., Huber, J.C. and Cook, R.L. (1987), "Geographic Differences and the Location of New Manufacturing Facilities," Journal of Urban Economics, Vol. 21, No. 1, pp. 83-104.

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INVENTORY MODEL IN A FUZZY ENVIRONMENT WITH ITS ASSOCIATED COSTS IN EXPONENTIAL MEMBERSHIP FUNCTIONS

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ABSTRACT

In this paper, an EOQ Model has been analyzed in a fuzzy environment. Its objective goal, holding cost, shortage cost, set-up cost and storage area are all represented by means of exponential membership functions. Fuzzy non-linear programming (FNLP) method with Lagrange multipliers is applied. This method has been illustrated by means of numerical examples. The variations of the results with those of the crisp model have been pointed out. Sensitivity analysis of the model has been carried out.

KEYWORDS

Fuzzy non–linear programming methods, Inventory management, Lagrange multipliers, Membership functions, Sensitivity analysis.

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1. INTRODUCTION

In the real world, keeping an inventory for future sale or use is very common in business. Often uncertainties may be common in business. Often uncertainties may be associated with demand, supply or various relevant costs. After the publication of classical lot-size formula by Harris in 1915, many researchers utilized EOQ model and currently these results are available in reference books and survey papers such as Raymond [11], Hadley and Whitin [5], Clark [3] and Simmons [13].

In a realistic situation, total expenditure for an inventory model may be limited. The inventory costs, holding, shortage and set-up costs and available warehouse space to store the product may be flexible with some vagueness for their values. All these parameters in any inventory model are normally variable uncertain, imprecise and adoptable to the optimum decision making process and the determination of optimum order quantity becomes a non-stochastic vague decision making process. The vagueness pertained in the above parameters induces to analyze the inventory problem in a fuzzy environment.

The early work in using fuzzy concept in decision making has been performed by Bellman and Zadeh [1] through introducing fuzzy goals, costs and constraints. The fuzzy linear programming model was formulated and an approach for solving linear programming model with fuzzy members has been presented by Zimmermann [15]. Hamacher.H, Leberling.H and Zimmermann.H.J [6] carried out the sensitivity analysis in fuzzy linear programming where Dutta.D, J.R.Rao and R.N.Tiwari [4] investigated the effect of tolerances in fuzzy linear and linear fractional programming. In 1995, T.K.Roy and M.Maiti [12] presented an EOQ model with constraint in a fuzzy environment. The model is solved by fuzzy non-linear programming model (FNLP) using Lagrange multipliers and illustrated with numerical examples. The fuzzy goal, storage area and costs are represented by exponential membership functions.

2. MATHEMATICAL ANALYSIS

A crisp non-linear programming problem may be defined as follows:

Min $g_0(X, C_0)$

Subject to:

$g_i(X, C_i) \leq b_i$, $i = 1,2, \dots \dots \dots . m$ (2.1)

$X \geq 0,$

where $X=(X_1, X_2, \dots \dots . . X_n)^T$ is a variable vector. g_0, g_i's , $i = 1,2, \dots \dots \dots . m$ are algebraic expressions in X with coefficients $C_0 \equiv (C_{01}, C_{02}, \dots \dots . , C_{0n})$ and $(C_{i1}, C_{i2}, \dots \dots . , C_{in})$ $i = 1,2, \dots \dots \dots . m$ respectively.

Introducing fuzziness in the crisp parameters, the system (2.1) in a fuzzy environment is:

$\widetilde{\text{Min}}\ g_0(X, \widetilde{C_0})$

Subject to:

$g_i(X, \tilde{C}_i) \leq \tilde{b}_i$, $i = 1,2, \dots \dots \dots . m$ (2.2)

$X \geq 0,$

where the wave bar (~) represents the fuzzification of the parameters.

In fuzzy set theory, the fuzzy objective, coefficients and constraints are defined by their membership functions which may be linear or non-linear. According to Bellman and Zadeh [1] and following Carlson and Korhonen [2] and Trappey, J.F.C., Liu, C.R. and Chang, T.C [14], problem (2.2) is transformed to

Max α

Subject to:

$g_i(X, {\mu_{\tilde{C}_i}}^{-1}(\alpha)) \leq {\mu_{\tilde{b}_i}}^{-1}(\alpha)$, $i = 0,1,2, \dots \dots \dots . m$ (2.3)

$X \geq 0,$

where membership functions of fuzzy coefficients are

$\mu_{\tilde{C}_i}(X) = (\mu_{\tilde{C}_{i1}}(X), \mu_{\tilde{C}_{i2}}(X), \dots\dots\dots \mu_{\tilde{C}_{in}}(X))$ and those of fuzzy objective and fuzzy constraints are

$\mu_{\tilde{b}_i}(X), i = 0,1,2, \dots \dots \dots . . m.$

Here, α is an additional variable which is known as aspiration level.

Therefore, the Lagrangian function is

$L(\alpha, X, \lambda) = \alpha - \sum_{i=0}^{m} \lambda_i (g_i(X, \mu_{\tilde{C}_i}{}^{-1}(\alpha)) - \mu_{\tilde{b}_i}{}^{-1}(\alpha))$

where $\lambda \equiv (\lambda_0, \lambda_1 \lambda_2, \ldots \ldots \lambda_m)^T$ are Lagrangian multiplier vectors.

According to the Kuhn-Tucker [10] necessary conditions, the optimal values

$(X_1^*, X_2^*, \ldots\ldots X_n^*, \lambda_0^*, \lambda_1^*, \ldots\ldots\ldots \lambda_m^*, \alpha^*)$

should satisfy

$\frac{\partial L}{\partial X_j} = 0,$

$\frac{\partial L}{\partial \alpha} = 0,$

$\lambda_i (g_i(X, \mu_{\tilde{C}_i}{}^{-1}(\alpha)) - \mu_{\tilde{b}_i}{}^{-1}(\alpha)) = 0$ (2.4)

$g_i(X, \mu_{\tilde{C}_i}{}^{-1}(\alpha)) - \mu_{\tilde{b}_i}{}^{-1}(\alpha) \leq 0$ and

$\lambda_i \leq 0, i = 0,1,2, \ldots \ldots m,$ j=1,2,3.......n

The Kuhn–Tucker[7] sufficient condition demand, that the objective function (for maximization) and constraints should be respectively concave and convex. Now solving equation (2.4), the optimal solution for the fuzzy non-linear programming problem is obtained.

3. EOQ MODEL WITH FUZZY COSTS, GOAL AND STORAGE AREA

In a crisp EOQ Model, the problem is to choose the order level Q(>0) which minimizes the average total cost C(Q) per unit time. That is

Min $C(Q) = \frac{1}{2} C_1 \left(\frac{Q_1{}^2}{Q}\right) + \frac{1}{2} C_2 \left(\frac{Q_2{}^2}{Q}\right) + C_3 \left(\frac{D}{Q}\right)$ -----------------------------(3.1)

Subject to:

$AQ_1 \leq B$

$Q_1 > 0$

where

C_1- Holding cost per unit time per unit quantity

C_2- Shortage cost per unit time per unit quantity

C_3- Set-up cost per period

D-Demand per unit time

A-The space required by each unit(in sq. mt)

B-Maximum available warehouse space(in sq mt)

When the above objective goal, costs and available storage area became fuzzy, the said problem is transformed to,

$\widetilde{Min}\ C(Q) = \frac{1}{2} \widetilde{C_1} \left(\frac{Q_1{}^2}{Q}\right) + \frac{1}{2} \widetilde{C_2} \left(\frac{Q_2{}^2}{Q}\right) + \widetilde{C_3} \left(\frac{D}{Q}\right)$ -----------------------------(3.2)

Subject to:

$AQ_1 \leq \tilde{B}$

$Q_1 > 0$

The corresponding fuzzy non-linear programming problem is,

Max α

Subject to:

$\frac{1}{2} \mu_{\widetilde{C_1}}{}^{-1}(\alpha) \left(\frac{Q_1{}^2}{Q}\right) + \frac{1}{2} \mu_{\widetilde{C_2}}{}^{-1}(\alpha) \left(\frac{Q_2{}^2}{Q}\right) + \mu_{\widetilde{C_3}}{}^{-1}(\alpha) \left(\frac{D}{Q}\right) \leq \mu_{\widetilde{C_0}}{}^{-1}(\alpha)$ ---------------------- (3.3)

$AQ_1 \leq \mu_{\tilde{B}}{}^{-1}(\alpha)$

$Q_1 > 0, \quad \alpha \in [0,1]$

where $\mu_{\widetilde{C_1}}(x), \mu_{\widetilde{C_2}}(x), \mu_{\widetilde{C_3}}(x), \mu_{\widetilde{C_0}}(x)$ and $\mu_{\tilde{B}}(X)$ are membership functions for fuzzy holding cost, shortage cost, set-up cost, objective goal and storage area respectively.

Here the Lagrangian function is

$L(\alpha, Q_1, Q_2, \lambda) = \alpha - \lambda_1 \left[\frac{1}{2} \mu_{\widetilde{C_1}}{}^{-1}(\alpha) \left(\frac{Q_1{}^2}{Q_1+Q_2}\right) + \frac{1}{2} \mu_{\widetilde{C_2}}{}^{-1}(\alpha) \left(\frac{Q_2{}^2}{Q_1+Q_2}\right) + \mu_{\widetilde{C_3}}{}^{-1}(\alpha) \left(\frac{D}{Q_1+Q_2}\right) - \mu_{\widetilde{C_0}}{}^{-1}(\alpha)\right] - \lambda_2 [AQ_1 - \mu_{\tilde{B}}{}^{-1}(\alpha)]$ --------(3.4)

where $Q = Q_1 + Q_2$ and λ_1, λ_2 are Lagrangian multipliers.

4. FUZZY GOAL, COSTS AND STORAGE AREA REPRESENTED BY EXPONENTIAL MEMBERSHIP FUNCTIONS

In this case

$$\mu_{\tilde{C}_i}(x) = \begin{cases} 1 & for \quad x > C_i \\ \dfrac{\left(q^{\frac{t(C_i - x)}{P_i}} - q^t\right)}{(1 - q^t)} & for\ C_i - P_i \leq x \leq C_i \\ 0 & for\ x < C_i - P_i \end{cases}$$

0<q<1, t>0, i=1,2,3

$$\mu_{\tilde{C}_0}(x) = \begin{cases} 1 & for \quad x < C_0 \\ \left(\dfrac{q^{\frac{t(x - C_0)}{P_0}} - q^t}{(1 - q^t)}\right) & for\ C_0 \leq x \leq C_0 + P_0 \\ 0 & for\ x > C_0 + P_0 \end{cases}$$

and

$$\mu_{\tilde{B}}(x) = \begin{cases} 1 & for \quad x < B \\ \dfrac{\left(q^{\frac{t(x - B)}{P}} - q^t\right)}{(1 - q^t)} & for \quad B \leq x \leq B + P \\ 0 & for \quad x > B + P \end{cases}$$

Here P_0, P and P_i's , (i=1,2,3) are the maximally acceptable violation of the aspiration levels C_0, B and C_i's , (i=1,2,3).

Considering the nature of these parameters, we assume membership functions to be non-decreasing for fuzzy inventory costs and non-increasing for fuzzy goal and storage area.

When q=0.5 and t=1

$\mu_{\tilde{C}_i}{}^{-1}(\alpha) = C_i - P_i \log_{0.5}(0.5 + 0.5\alpha)$, $\quad i = 1,2,3$

$\mu_{\tilde{C}_0}{}^{-1}(\alpha) = C_0 + P_0 \log_{0.5}(0.5 + 0.5\alpha)$ and

$\mu_{\tilde{B}}^{-1}(\alpha) = B + P\log_{0.5}(0.5 + 0.5\alpha)$

The Lagrangian Function is

$$L(\alpha, Q_1, Q_2, \lambda_1, \lambda_2) = \alpha - \lambda_1\Big[\tfrac{1}{2}(C_1 - P_1\log_{0.5}(0.5+0.5\alpha))\Big(\tfrac{Q_1^2}{Q_1+Q_2}\Big) + \tfrac{1}{2}(C_2 - P_2\log_{0.5}(0.5+0.5\alpha))\Big(\tfrac{Q_2^2}{Q_1+Q_2}\Big) + (C_3 - P_3\log_{0.5}(0.5+0.5\alpha))\Big(\tfrac{D}{Q_1+Q_2}\Big) - (C_o + P_0\log_{0.5}(0.5+0.5\alpha))\Big] - \lambda_2[AQ_1 - (B + P\log_{0.5}(0.5+0.5\alpha))]$$

---------(4.1)

$$= \alpha - \lambda_1\Big[\tfrac{1}{2}\Big(C_1 - \tfrac{P_1\log(0.5+0.5\alpha)}{\log\frac{1}{2}}\Big)\Big(\tfrac{Q_1^2}{Q_1+Q_2}\Big) + \tfrac{1}{2}\Big(C_2 - \tfrac{P_2\log(0.5+0.5\alpha)}{\log\frac{1}{2}}\Big)\Big(\tfrac{Q_2^2}{Q_1+Q_2}\Big) + \Big(C_3 - \tfrac{P_3\log(0.5+0.5\alpha)}{\log\frac{1}{2}}\Big)\Big(\tfrac{D}{Q_1+Q_2}\Big) - \Big(C_o + \tfrac{P_0\log(0.5+0.5\alpha)}{\log\frac{1}{2}}\Big)\Big] - \lambda_2\Big[AQ_1 - \Big(B + P\tfrac{\log(0.5+0.5\alpha)}{\log\frac{1}{2}}\Big)\Big]$$

--------(4.2)

The Kuhn-Tucker necessary conditions are

$$\frac{\partial L}{\partial \alpha} = 0 \Rightarrow = 1 + \frac{\lambda_1}{2}\frac{P_1}{\log\frac{1}{2}}\frac{1}{(1+\alpha)}\Big(\frac{Q_1^2}{Q_1+Q_2}\Big) + \frac{\lambda_1}{2}\frac{P_2}{\log 1/2}\frac{1}{(1+\alpha)}\Big(\frac{Q_2^2}{Q_1+Q_2}\Big) + \frac{\lambda_1 P_3}{\log 1/2}\frac{1}{(1+\alpha)}\Big(\frac{D}{Q_1+Q_2}\Big) + \frac{\lambda_1 P_0}{\log 1/2}\frac{1}{(1+\alpha)} + \frac{\lambda_2 P}{\log 1/2}\frac{1}{(1+\alpha)} = 0 \quad \text{---------(4.3)}$$

$$\frac{\partial L}{\partial \lambda_1} = 0 \Rightarrow \begin{bmatrix} \frac{1}{2}\Big(C_1 - \frac{P_1\log(0.5+0.5\alpha)}{\log\frac{1}{2}}\Big)\Big(\frac{Q_1^2}{Q_1+Q_2}\Big) + \frac{1}{2}\Big(C_2 - \frac{P_2\log(0.5+0.5\alpha)}{\log\frac{1}{2}}\Big)\Big(\frac{Q_2^2}{Q_1+Q_2}\Big) + \\ \Big(C_3 - \frac{P_3\log(0.5+0.5\alpha)}{\log 1/2}\Big)\Big(\frac{D}{Q_1+Q_2}\Big) - \Big(C_o + \frac{P_0\log(0.5+0.5\alpha)}{\log 1/2}\Big) \end{bmatrix} = 0$$

---------(4.4)

$$\frac{\partial L}{\partial \lambda_2} = 0 \Rightarrow AQ_1 - \Big(B + \frac{P\log(0.5+0.5\alpha)}{\log 1/2}\Big) = 0 \quad \text{-------- (4.5)}$$

$$\frac{\partial L}{\partial Q_1} = 0 \Rightarrow \frac{\lambda_1}{2}\Big(C_1 - \frac{P_1\log(0.5+0.5\alpha)}{\log 1/2}\Big)\Big(\frac{Q_1^2+2Q_1Q_2}{(Q_1+Q_2)^2}\Big) - \frac{\lambda_1}{2}\Big(C_2 - \frac{P_2\log(0.5+0.5\alpha)}{\log 1/2}\Big)\Big(\frac{Q_2^2}{(Q_1+Q_2)^2}\Big) - \lambda_1\Big(C_3 - \frac{P_3\log(0.5+0.5\alpha)}{\log\frac{1}{2}}\Big)\Big(\frac{D}{(Q_1+Q_2)^2}\Big) + \lambda_2 A = 0 \quad \text{-----(4.6)}$$

$$\frac{\partial L}{\partial Q_2} = 0 \Rightarrow \frac{\lambda_1}{2}\Big(C_1 - \frac{P_1\log(0.5+0.5\alpha)}{\log 1/2}\Big)\Big(\frac{Q_1^2}{(Q_1+Q_2)^2}\Big) - \frac{\lambda_1}{2}\Big(C_2 - \frac{P_2\log(0.5+0.5\alpha)}{\log 1/2}\Big)\Big(\frac{Q_2^2+2Q_1Q_2}{(Q_1+Q_2)^2}\Big) + \lambda_1\Big(C_3 - \frac{P_3\log(0.5+0.5\alpha)}{\log 1/2}\Big)\Big(\frac{D}{(Q_1+Q_2)^2}\Big) = 0 \quad \text{-------- (4.7)}$$

Therefore, $AQ_1 = B + \dfrac{P\log(0.5+0.5\alpha)}{\log 1/2}$

$$Q_1^* = \frac{\Big(B + \frac{P\log(0.5+0.5\alpha)}{\log 1/2}\Big)}{A} \quad \text{-------- (4.8)}$$

The optimal order quantity is

$$Q_1^* = \frac{(B + P\log_{0.5}(0.5+0.5\alpha))}{A} \quad \text{--------- (4.9)}$$

and the optimum Re-order level Q_2^* is

$$Q_2^* = \Big[\frac{C_1 - P_1\log_{0.5}(0.5+0.5\alpha)}{C_2 - P_2\log_{0.5}(0.5+0.5\alpha)}\Big] Q_1^* \quad \text{--------- (4.10)}$$

where α^* is a root of

$$(C_1 - P_1\log_{0.5}(0.5+0.5\alpha))\,(B + P\log_{0.5}(0.5+0.5\alpha))^2 + (C_2 - P_2\log_{0.5}(0.5+0.5\alpha))\,Q_2^2A^2 + 2DA^2(C_3 - P_3\log_{0.5}(0.5+0.5\alpha)) - 2A(C_0 + P_0\log_{0.5}(0.5+0.5\alpha)) + (B + P\log_{0.5}(0.5+0.5\alpha)) = 0 \quad \text{-------- (4.11)}$$

Remark 4.1

If the fuzzy goal and storage area are represented by means of linear membership functions and costs by exponential membership functions, then the equation (4.11) reduces to

$$(C_1 - P_1\log_{0.5}(0.5+0.5\alpha))\,(B + (1-\alpha)P)^2 + (C_2 - P_2\log_{0.5}(0.5+0.5\alpha))\,Q_2^2A^2 + 2(C_3 - P_3\log_{0.5}(0.5+0.5\alpha))DA^2 - 2(C_o + (1-\alpha)P_0)QA^2 = 0 \quad \text{-------- (4.12)}$$

which is the equation derived in [9].

Remark 4.2

When $Q_2 = 0$, Then the equation (4.12) reduces to

$$(C_1 - P_1\log_{0.5}(0.5+0.5\alpha))\,(B + (1-\alpha)P)^2 + 2(C_3 - P_3\log_{0.5}(0.5+0.5\alpha))DA^2 - 2(C_o + (1-\alpha)P_0)(B + (1-\alpha)P)A = 0 \quad \text{-------- (4.13)}$$

which is the equation derived in [12].

5. NUMERICAL RESULTS FOR BOTH CRISP AND FUZZY MODEL

To illustrate the above fuzzy models and corresponding crisp model, let $C_1 =$ ₹ 5, $P_1 =$ ₹ 2, $C_2 =$ ₹ 25, $P_2 =$ ₹ 9, $C_3 =$ ₹ 100, $P_3 =$ ₹ 50, $C_0 =$ ₹ 1700, $P_0 =$ ₹ 300, $D = 5000$ units, $B = 150$ sq. mt, $A = 0.5$ sq. mt, $P = 50$ sq. mt and the results are:

In the table – I the optimal values are given for these fuzzy models with different combinations of membership functions along with the crisp model. Here the amount of expenditure for the crisp models varies from ₹ 1520.83 to ₹ 2138.89.

As our permissible expenditure range is (₹ 1700, ₹ 2000), only one crisp model expenditure fall within this range, though this value with holding cost ₹ 3 is more than those of fuzzy models with holding costs respectively ₹ 4.53, 4.55, 4.56 and 4.57. Therefore, fuzzy analysis replaces the laborious and time consuming parametric studies and the optimum results are obtained easily. Now depending on the experience or from the past observed data, the exact form of membership functions for the variations of holding cost, set-up cost, permissible expenditure (objective goal) and storage area can be defined and then the optimum values are obtained for the appropriate fuzzy model.

Remark 5.1

Total average cost for the fuzzy model corresponding to this model – 4 is slightly higher than the total average cost for the fuzzy model corresponding to the model – 4 in [9].

6. SENSITIVITY

Following Dutta.D, J.R.Rao and R.N.Tiwari [4], we study the effective of tolerances[8] in the proposed model – 4 with the earlier numerical values and construct the following five Tables (TABLES – II, III, IV, V, VI) for the effect of variations in P_0, P_1, P_2, P_3 and P respectively.

From the above tables, we make the following observations

1. In Table – II, when P_0 is about 300, 5% changes in P_0 induces 0.04 %, 0.03 %, 0.08 % and 0.23 % changes in $Q_1^*, Q_2^*, C^*(Q^*)$ and α^* respectively. Moreover as P_0 increases, α^* increases but it never becomes 1 as it is expected. Here Q^* becomes almost invariant and costs attain their highest allowable values for the large values of P_0. Average cost also increases with the increase of tolerance of objective goal.
2. In Table – III, when P_1 is about 2, $Q_1^*, Q_2^*, C^*(Q^*)$ and α^* are changes by 0.07%, 0.55 %, 0.01% and 0.39% respectively for 5% changes in P_1. Set-up cost increases with the decrease of holding cost, shortage cost increases with the decrease of holding cost and as a result, average cost decreases with the decrease of holding cost's tolerance.

3. In Table – IV, when P_2 is about 2, ${Q_1}^*$, ${Q_2}^*$, $C^*(Q^*)$ and α^* are changes by 0.06 % , 0.41 %, 0.04 % and 0.36 % respectively for 5 % change in P_2. Set-up cost increases with the decrease of holding cost, shortage cost decreases with the decrease of holding cost and as a result, average cost decreases with the decrease of holding cost's tolerance.
4. In Table – V, when P_3 is approximately 50, there is a change in ${Q_1}^*$, ${Q_2}^*$, $C^*(Q^*)$ and α^* by 0.16 % , 0.13 % , 0.08 % and 0.86 % respectively for the change of P_3 by 5%. Here holding cost increases with decrease of set-up cost whereas the average cost decreases with the less tolerances of set-up cost, shortage cost increases with the increase of holding cost
5. In Table – VI, when P is about 50, its 5% change brings 0.34 % , 0.34 % , 0.44 % and 0.1 % change in ${Q_1}^*$, ${Q_2}^*$, $C^*(Q^*)$ and α^* respectively. Here average cost decreases with increased tolerances of storage area. Set-up cost decreases with the increase of holding cost and shortage cost increases with the increase of holding cost.

7. CONCLUSION

In this paper, it is proposed a real-life inventory problem in a fuzzy environment and presented its solutions along with a sensitivity analysis approach. The proposed methodology can be extended to other inventory problem including the ones with storages, shortages, deteriorating items, etc. This model has been developed for multi-item inventory problems under several limitations such as limitation inventory levels.

8. REFERENCES

1. Bellman R.E, Zadeh L.A., "Decision making in a fuzzy environment Management Sciences", 1970, Volume 17, pp.141 – 164.
2. Carlson,C., and Korhonen,P., "A parametric Approach to Fuzzy Linear programming", Fuzzy sets and systems, 1986, Vol.20, pp.17-30.
3. Clark.A.J, "An informal survey of multi-echelon inventory theory". Naval Research Logistics Quarterly 19, 1972, pp.621-650.
4. Dutta.D, J.R.Rao and R.N.Tiwari, "Effect of tolerance in fuzzy linear fractional programming", Fuzzy sets and systems, 55, 1993, pp 133-142.
5. Hadley.G and Whitin.T.M, "Analysis of Inventory Systems", Printice-Hall, Englewood Cliffs, NJ. 1958.
6. Hamacher.H, Leberling.H and Zimmermann.H.J, "Sensitivity analysis in fuzzy linear programming", Fuzzy sets and systems1, 1978, pp.269-281.
7. Hanson M.A., "On sufficiency of the Kuhn-Tucker conditions", Journal of Mathematical Analysis and Applications, 1981, pp. 545-550.
8. Kadambavanam.K and Punniakrishnan.K. "Fuzzy Inventory Model Allowing Shortages and Storage Constraint" International Journal of Production Technology and Management Research, New Delhi, Vol.2 , No.2 , pp.95-101 , July-Dec 2011.
9. Kadambavanam.K and Punniakrishnan.K. "Inventory Model in a Fuzzy Environment with Exponential Membership Cost Functions", CiiT International Journal of Fuzzy Systems, Coimbatore, Vol.3, No.9, pp.383-390, (2011).
10. Kuhn.H.W and Tucker.A.W. "Non-linear programming". In.J.Neyman (ed), proceedings second Berkely symposium on mathematical statistics and probability, University of california pracil, 1951, pp.481-498.
11. Raymond.F.E, "Quantity and Economic in Manufacturer", Mc Graw Hill Book Co; New York. 1931.
12. Roy.T.K. and Maiti.M., 1995. "A fuzzy inventory model with constraint". Opsearch, Volume 32, No.4, pp.287-298.
13. Simmons, D.M., "Non-linear programming for Operations Research", prentice-Hall, Englewood, clifts, N.J.,1975.
14. Trappey, J.F.C.,Liu, C.R. and Chang, T.C., "Fuzzy non-linear programming. Theory and applications in manufacturing", Int. J. Production Research, 1988, Vol 26, No.5, 975-985.
15. Zimmermann.H.J., "Fuzzy set theory and its applications". Allied publishers Limited, New Delhi in association with Kluwar Academic publishers 2nd Revised Edition, Boston. 1996.

TABLES

TABLE I - COMPARISON TABLE

Model	Holding Cost C_1 (Rs.)	Shortage Cost C_2 (Rs.)	Set-up Cost C_3 (Rs.)	Storage Area B (Sq.m.)	Optimum of Q_1*	Optimum of Q_2*	Average Total Cost C*(Q*) (Rs.)	Aspiration Level α*
Crisp Model	5	25	100	150	300	60	2138.89	
Crisp Model	5	25	100	200	400	80	2041.67	
Crisp Model	3	20	100	150	300	45	1899.28	
Crisp Model	3	20	100	200	400	60	1686.96	
Crisp Model	3	20	50	150	300	45	1174.64	
Crisp Model	3	20	50	200	400	60	1143.48	
Crisp Model	5	25	50	150	300	60	1444.44	
Crisp Model	5	25	50	200	400	80	1520.83	
Fuzzy Model	4.53	22.89	88.28	161.72	323.45	64.03	1871.91	0.70
Fuzzy Model	4.55	22.97	88.70	161.30	322.60	63.89	1881.11	0.71
Fuzzy Model	4.56	23.04	89.12	160.88	321.76	63.74	1890.28	0.72
Fuzzy Model	4.57	23.08	89.33	160.67	321.34	63.67	1894.86	0.725

TABLE II - EFFECT OF VARIATIONS IN P_0

P_0	α*	Q_1*	Q_2*	C_1*	C_2*	C_3*	C* (Q*)	B*
300	0.6985	323.57	64.05	4.53	22.88	88.21	1870.53	161.79
315	0.7001	323.44	64.03	4.53	22.89	88.28	1872.00	161.72
345	0.7054	322.99	63.95	4.54	22.93	88.51	1876.88	161.49
500	0.7266	321.21	63.65	4.58	23.09	89.40	1896.32	160.60
1000	0.7749	317.23	62.98	4.66	23.45	91.39	1940.23	158.61
5000	0.8851	308.54	61.49	4.83	24.23	95.73	2038.59	154.27
10000	0.9135	306.38	61.12	4.87	24.43	96.81	2063.58	153.19
50000	0.9419	304.25	60.75	4.91	24.62	97.87	2088.43	152.13
100000	0.9459	303.96	60.69	4.92	24.64	98.02	2091.92	151.98
500000	0.94917	303.71	60.65	4.93	24.67	98.14	2094.77	151.86

TABLE III - EFFECT OF VARIATIONS IN P_1

P_1	α*	Q_1*	Q_2*	C_1*	C_2*	C_3*	C* (Q*)	B*
0	0.6708	325.95	71.91	5.00	22.66	87.03	1908.58	162.97
1	0.6865	324.60	67.72	4.75	22.79	87.70	1889.30	162.30
2	0.7001	323.44	64.03	4.53	22.89	88.28	1872.00	161.72
2.5	0.7135	322.31	62.27	4.44	22.99	88.85	1871.03	161.15
3.2	0.7245	321.38	60.11	4.32	23.08	89.31	1864.03	160.69
4.3	0.7355	320.46	57.01	4.12	23.16	89.77	1849.20	160.23
5	0.7485	319.39	55.36	4.03	23.26	90.31	1848.56	159.69

TABLE - IV EFFECT OF VARIATIONS IN P_2

P_2	α*	Q_1*	Q_2*	C_1*	C_2*	C_3*	C* (Q*)	B*
0	0.6708	325.95	65.19	5.00	25.00	87.03	1927.36	162.97
1	0.689	324.38	62.32	4.76	24.76	87.81	1906.76	162.19
2	0.691	324.21	59.72	4.52	24.52	87.89	1876.69	162.11
2.5	0.7034	323.16	58.50	4.42	24.42	88.42	1872.72	161.58
3.2	0.7143	322.24	56.89	4.29	24.29	88.88	1863.10	161.12
4.3	0.7257	321.28	54.49	4.08	24.08	89.36	1845.20	160.64
5	0.7307	320.86	52.99	3.96	23.96	89.57	1832.68	160.43
6	0.7458	319.61	51.29	3.82	23.82	90.19	1826.86	159.81
7	0.7559	318.78	49.60	3.69	23.69	90.61	1817.27	159.39
8	0.7665	317.91	48.12	3.57	23.57	91.04	1810.70	158.96
9	0.7859	316.33	47.46	3.53	23.53	91.83	1820.47	158.17

TABLE - V EFFECT OF VARIATIONS IN P_3

P_3	α*	Q_1*	Q_2*	C_1*	C_2*	C_3*	C* (Q*)	B*
0	0.5254	339.08	66.58	4.22	21.48	100.00	1947.73	169.54
10	0.5941	332.73	65.56	4.35	22.05	96.73	1937.23	166.36
30	0.6885	324.43	64.19	4.51	22.80	92.67	1924.16	162.21
50	0.7001	323.44	64.03	4.53	22.89	88.28	1872.00	161.72
60	0.7242	321.41	63.69	4.57	23.07	87.16	1866.33	160.70
70	0.7352	320.49	63.53	4.59	23.16	85.66	1850.82	160.24
80	0.7445	319.72	63.40	4.61	23.23	84.23	1835.45	159.86
90	0.7654	318.00	63.11	4.64	23.38	83.80	1837.17	159.00
100	0.7974	315.41	62.67	4.69	23.61	84.59	1858.62	157.70

TABLE - VI EFFECT OF VARIATIONS IN P

P	α*	Q_1*	Q_2*	C_1*	C_2*	C_3*	C* (Q*)	B*
50	0.7001	323.44	64.03	4.53	22.89	88.28	1872.00	161.72
55	0.7014	325.66	64.47	4.53	22.90	87.17	1855.37	162.83
70	0.7115	331.46	65.64	4.55	22.98	84.27	1815.19	165.73
80	0.7234	334.36	66.25	4.57	23.07	82.82	1797.78	167.18
100	0.7345	341.10	67.61	4.59	23.15	79.45	1754.64	170.55
150	0.7555	356.44	70.71	4.62	23.31	71.78	1664.29	178.22
160	0.7846	352.61	70.03	4.67	23.52	73.70	1695.41	176.30

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